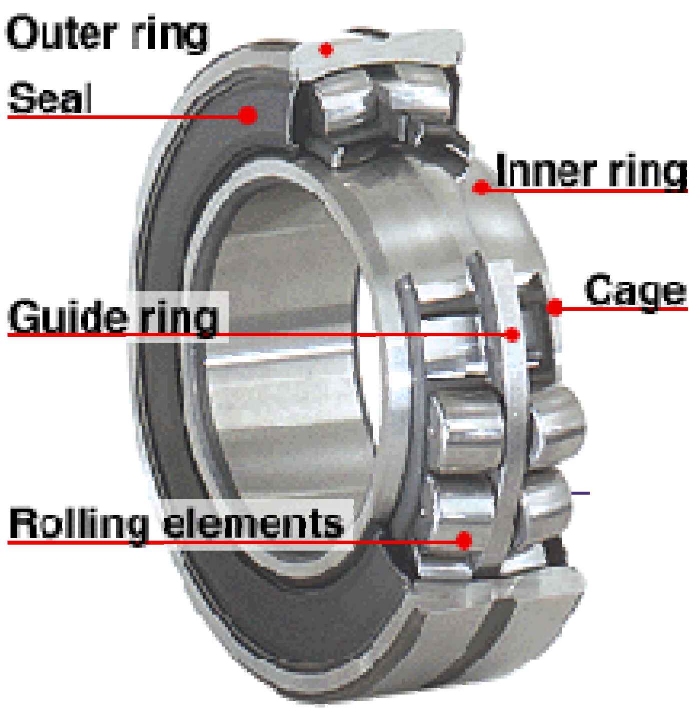
UNIVERSITY OF TWENTE.

THIN LAYER FLOW MODELING

PREDICTION OF FILM DECAY IN EHL CONTACTS AND ROLLING ELEMENT BEARINGS

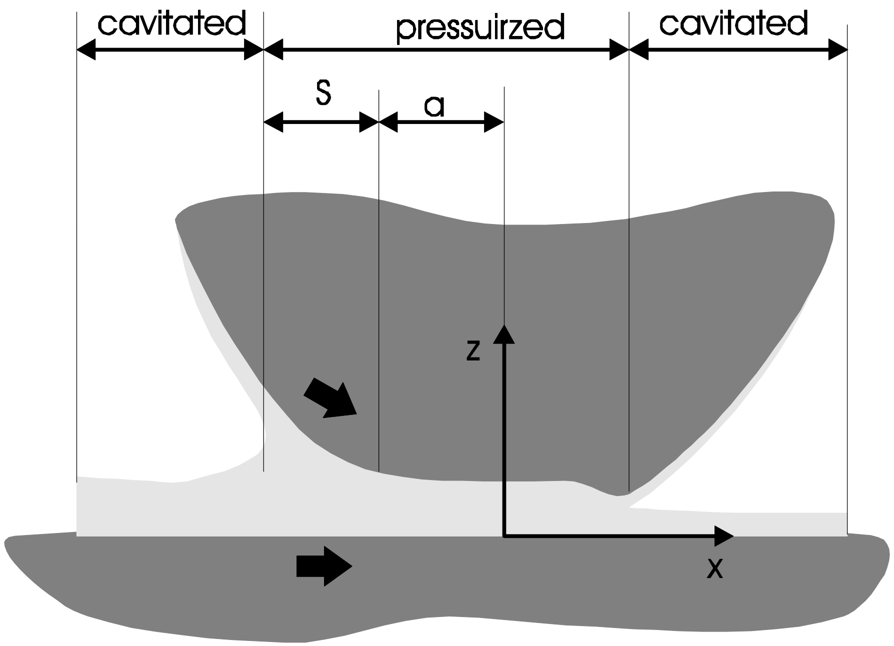
C.H. (KEES) VENNER

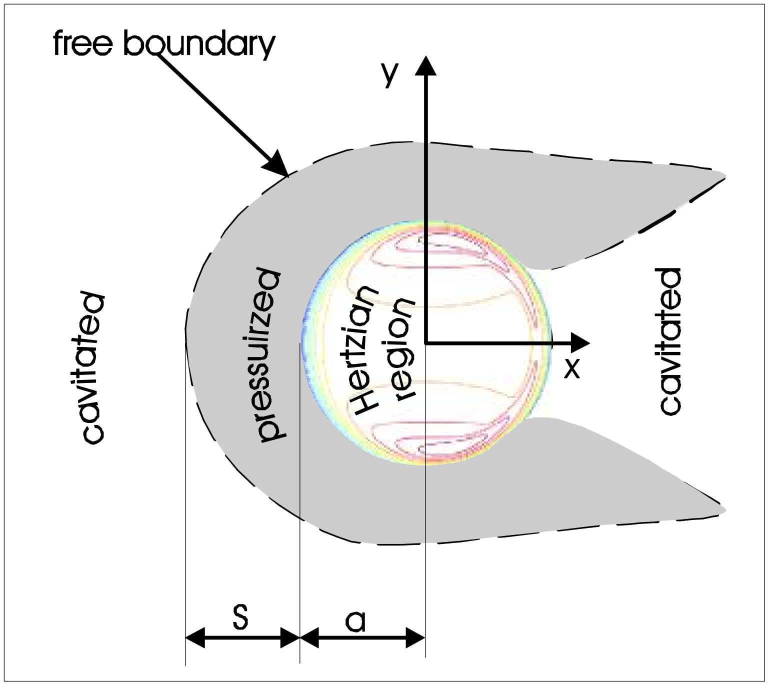
ROLLING ELEMENT BEARINGS



SINGLE CONTACT MODELLING (EHL)

Experimental





SINGLE CONTACT MODELING (EHL)

Flow: Navier Stokes, Narrow Gap assumption :

$$\frac{\partial}{\partial X}\left(e\,\frac{\partial P}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\frac{\partial P}{\partial Y}\right)-\Lambda(T)\frac{\partial\left(q\,\overline{r}\,H\right)}{\partial X}-\frac{\partial\left(q\,\overline{r}\,H\right)}{\partial T}=0$$

Gap height h: undeformed shape+elastic deformation

$$H(X,Y,T)=-\Delta(T)+\frac{X^2}{2}+\frac{Y^2}{2}+\frac{2}{p^2}\iint_S\frac{P(X',Y',T)dX'dY'}{\sqrt{(X-X')^2+(Y-Y')^2}}$$

Equation of Motion

$$\frac{1}{\Omega^2}\frac{d^2\Delta}{dT^2}+\frac{3}{2p}\iint_S P(X,Y,T)dXdY+\overline{K}\cdot\Delta=1+\overline{K}\Delta_\infty$$

MULTISCALE/MULTILEVEL COMPUTATIONAL METHODS

Conceptual approach:

- Identify **problematic** components responsible for computational slowness (slow convergence, multi-summations).
- Design **accurate** representation for **efficient** solution (computation)

- **Appearances:**

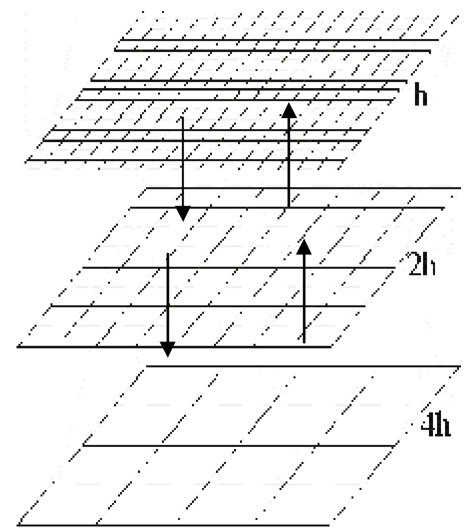
Standard: **Geometric** Multigrid

Advanced: General Systems: **AMG**

Advanced: Physics, Chemistry, Particles, etc.

GEOMETRIC MULTIGRID

- Iterative Process **bad solver** but **good smoother**
- Smooth error can accurately be approximated on **coarser grid**
- Solve error on coarser grid
- Correct fine grid solution
- Result: **grid independent** high convergence rate O(0.1), work O(N)



- **Geometric MG**: Fix coarsening and intergrid operators, design good smoother. Advantage: Principle straightforward, non linearity equally efficient. Disadvantage: Sometimes not trivial (stability for integral equations)
- **EHL** Elastic deformation integrals (Multilevel Multi-Integration)

ALGEBRAIC MULTIGRID (AMG)

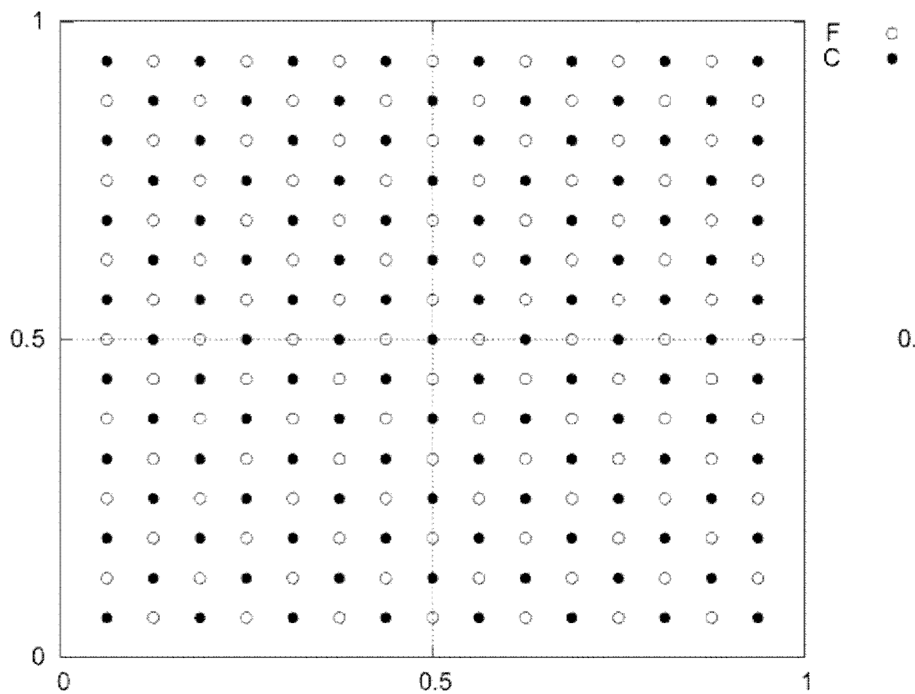
System of equations

$$\mathbf{Au=f}$$

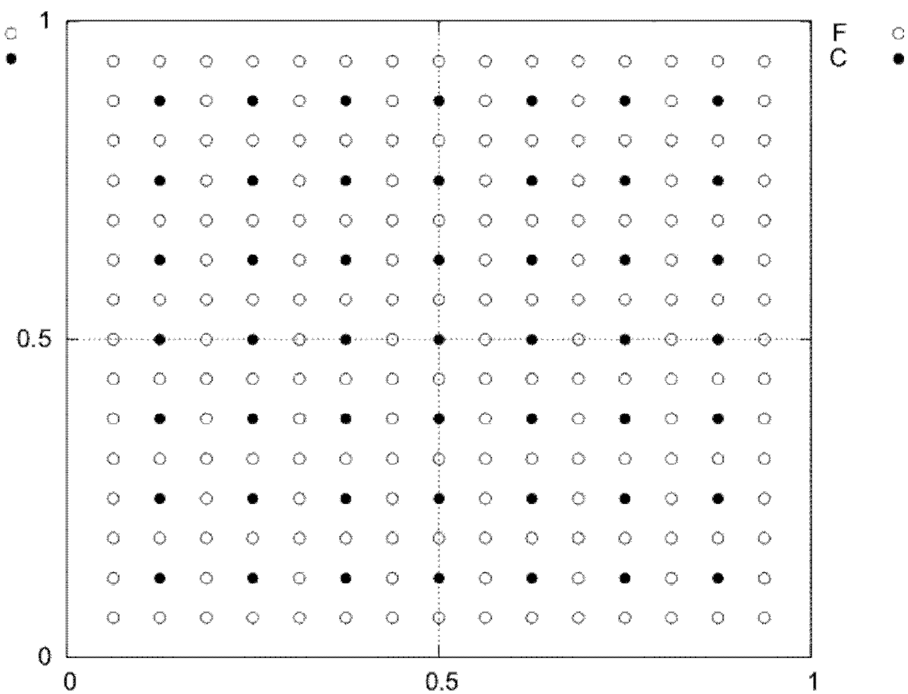
- **Fix** iterative scheme (GS relaxation, Kaczmarz relaxation)
- Matrix **A** and iterative scheme determine coarsening and intergrid operators such that slow to converge error is accurately approximated.

- Advantage: Little knowlede of system required, Very robust !
- Disadvantage: Set up more expensive (only once), Non-linearity more involved (but still no global linearization needed)

AMG: Example $\frac{\partial^2 u}{\partial x^2} + e\,\frac{\partial^2 u}{\partial y^2} = f(x, y)$

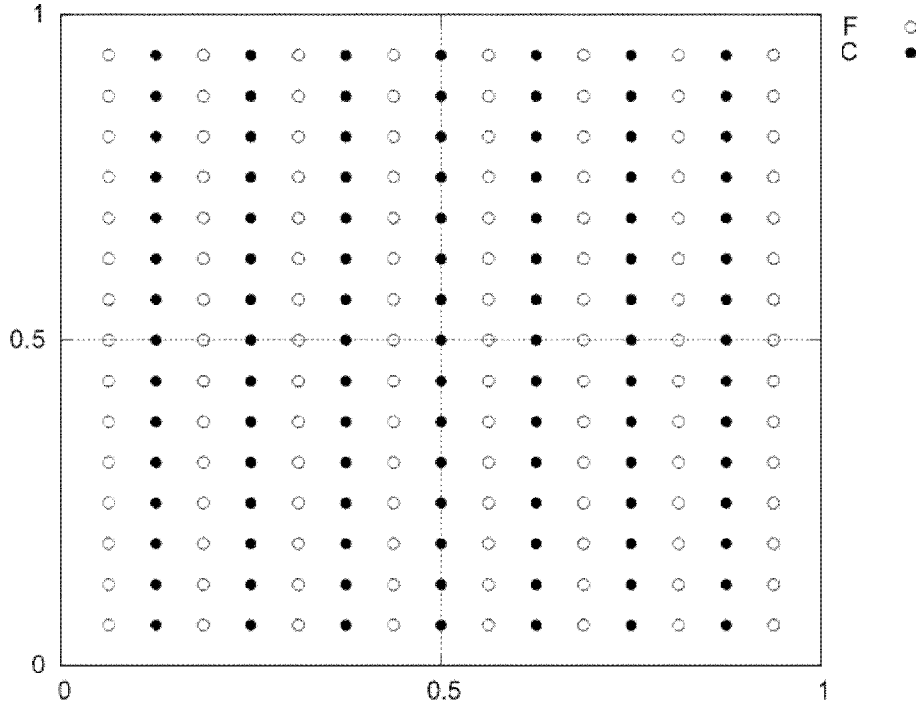


$e=1$ **5 point**

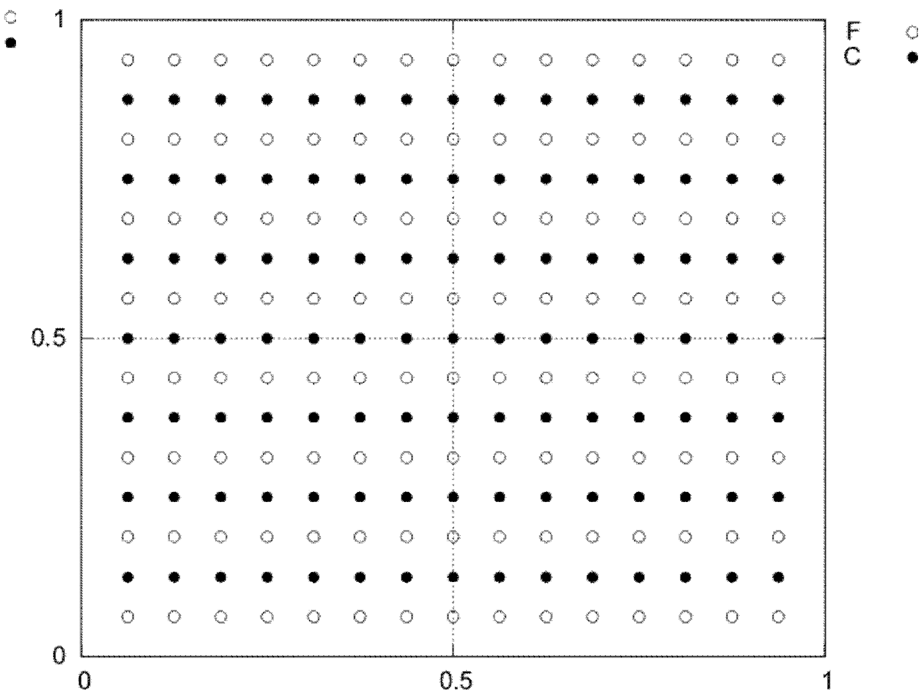


$e=1$ **5 point**

AMG: Example $\frac{\partial^2 u}{\partial x^2} + e\,\frac{\partial^2 u}{\partial y^2} = f(x,y)$

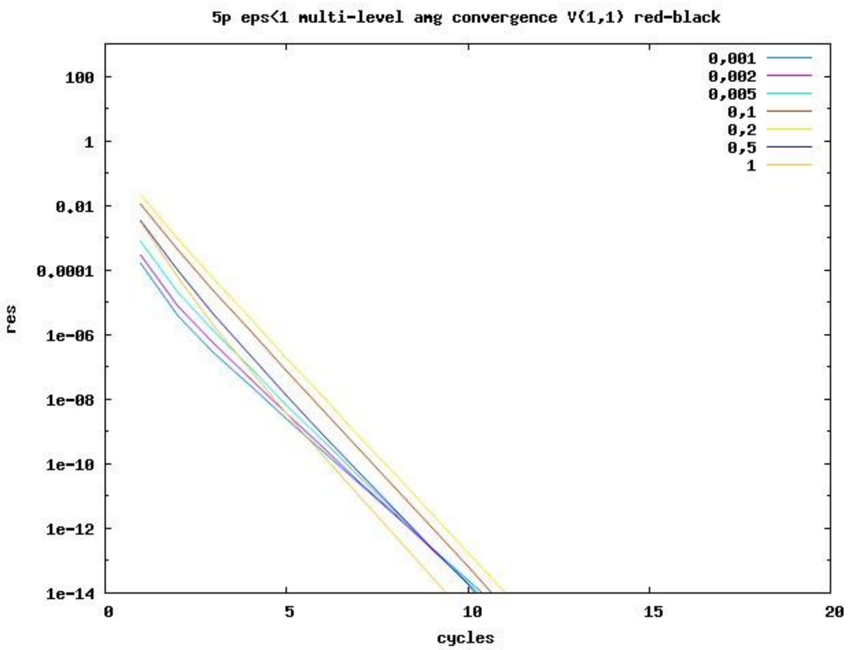


$e = 0$ **5 point**

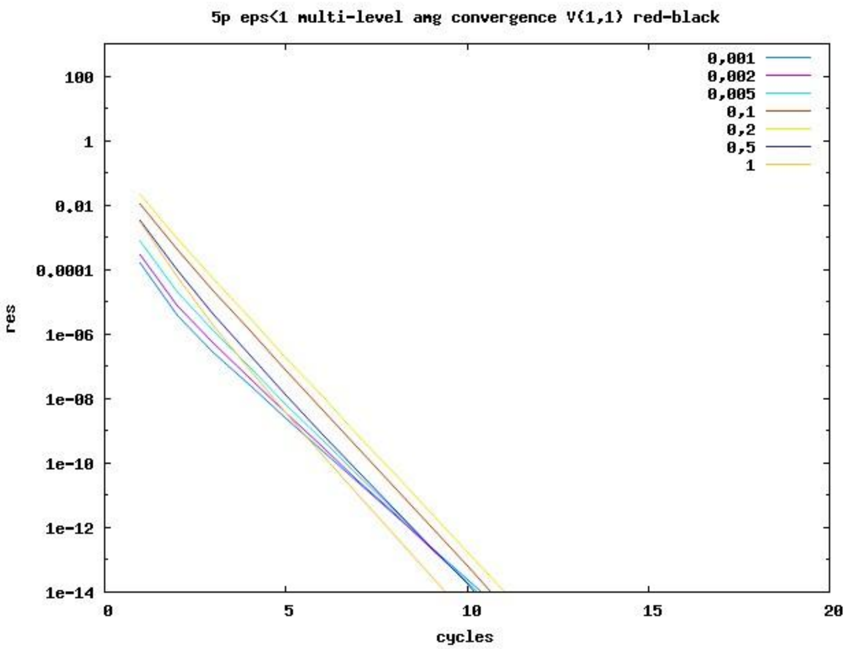


$e = 1000$ **5 point**

AMG: Example $$\frac{\partial^2 u}{\partial x^2} + e\,\frac{\partial^2 u}{\partial y^2} = f(x,y)$$

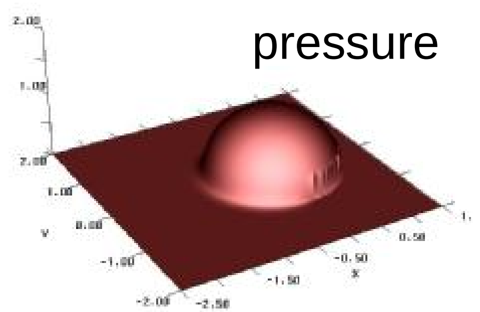


$e < 1$

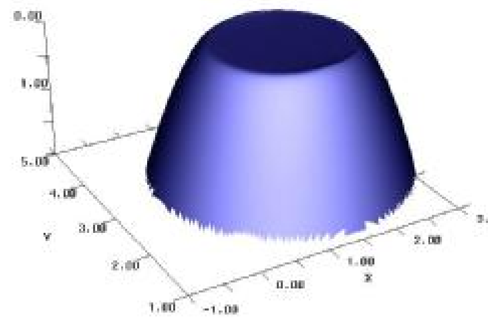


$e > 1$

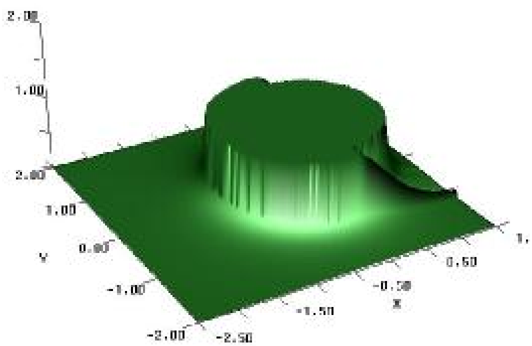
RESULTS SINGLE CONTACT EHL



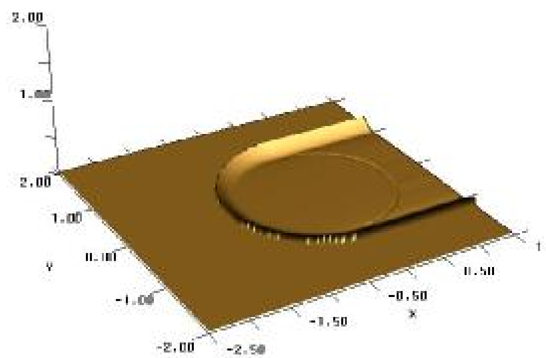
pressure



film

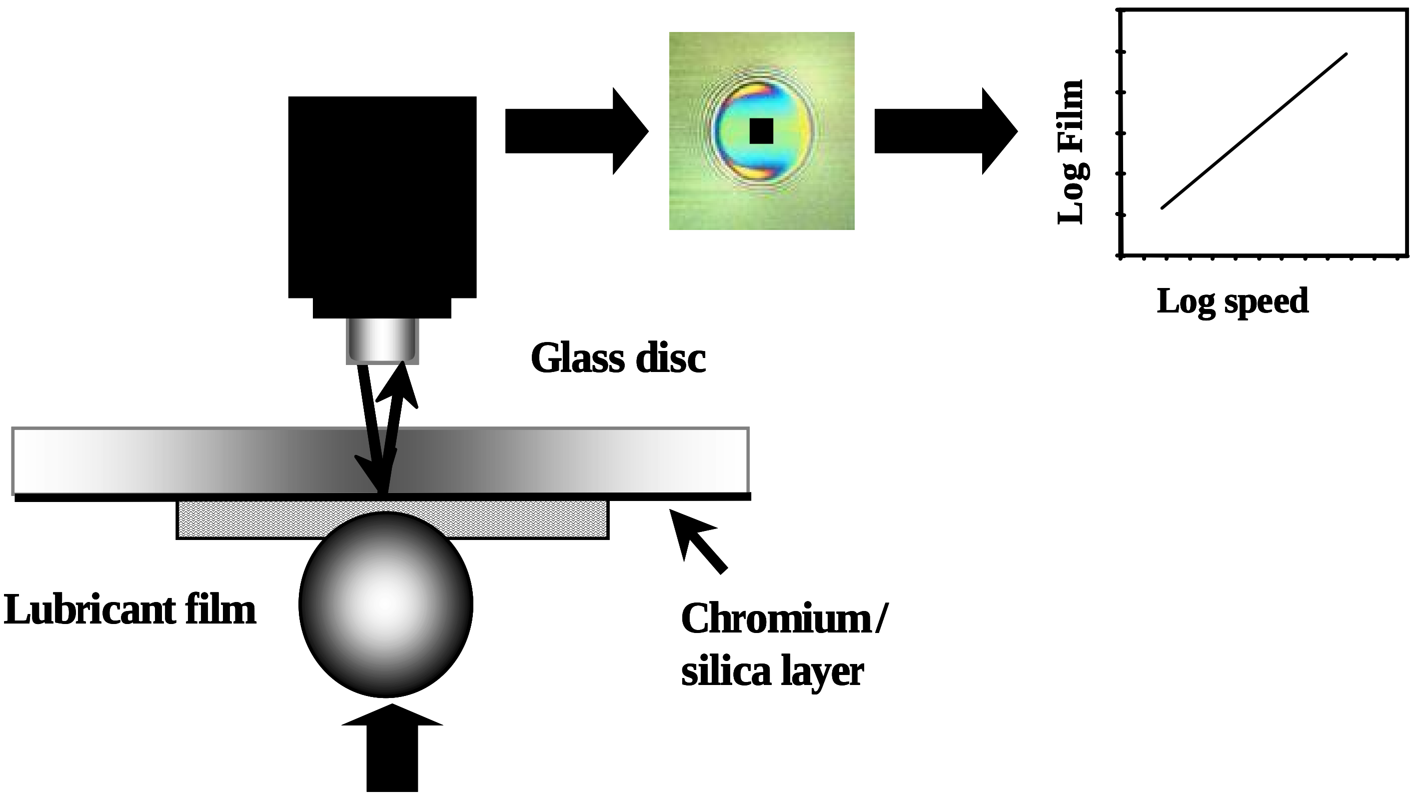


Fractional film content



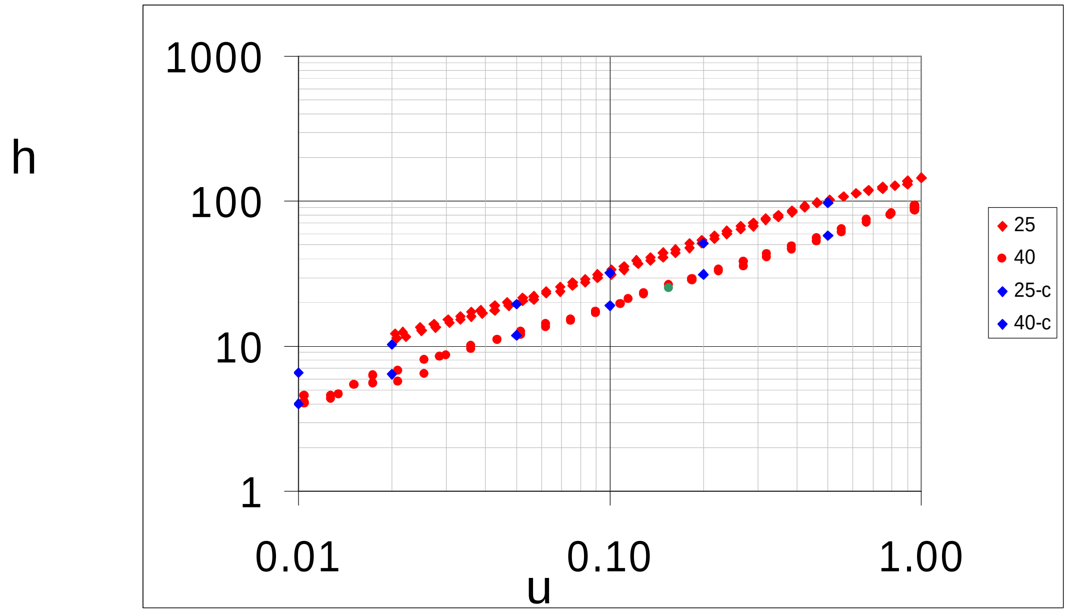
footprint

SINGLE CONTACT VALIDATION

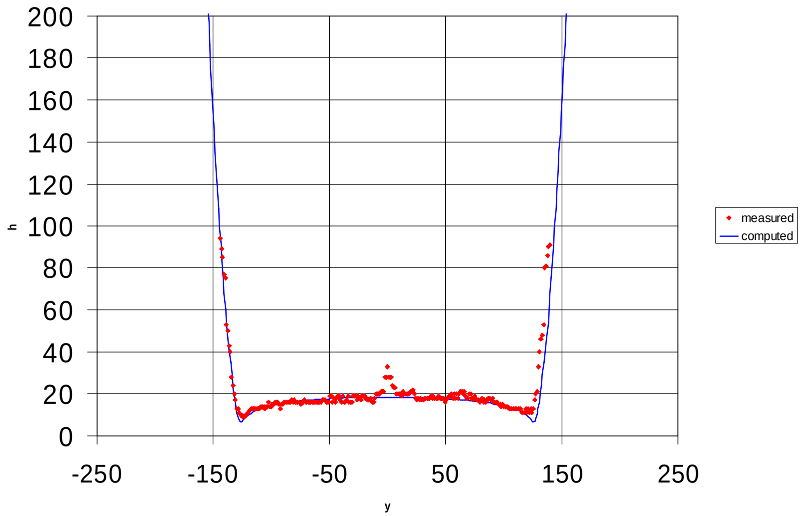


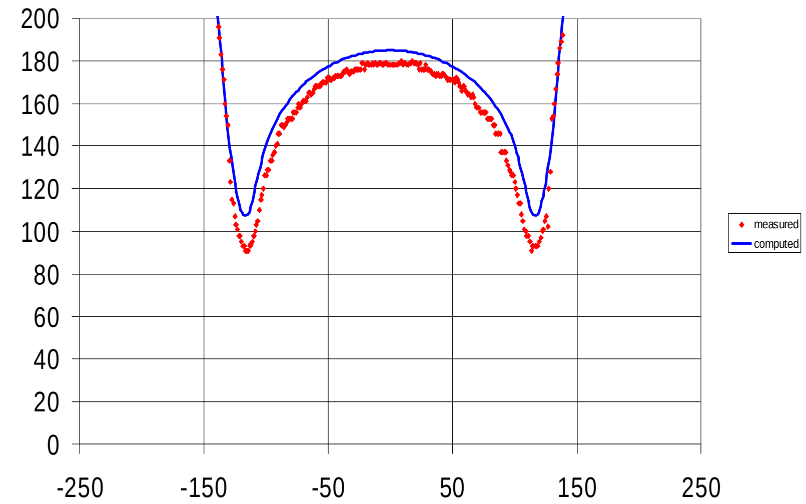
STEADY STATE

Standard mineral oil (shell TT9)

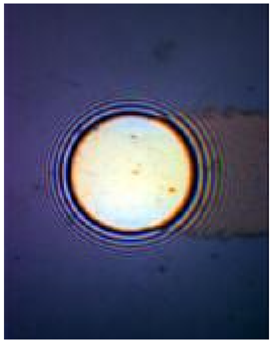


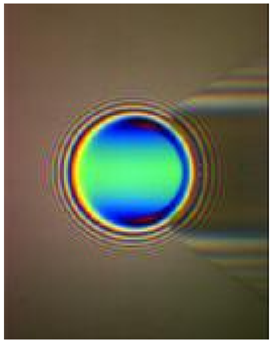
STEADY STATE





U=0.05 m/s

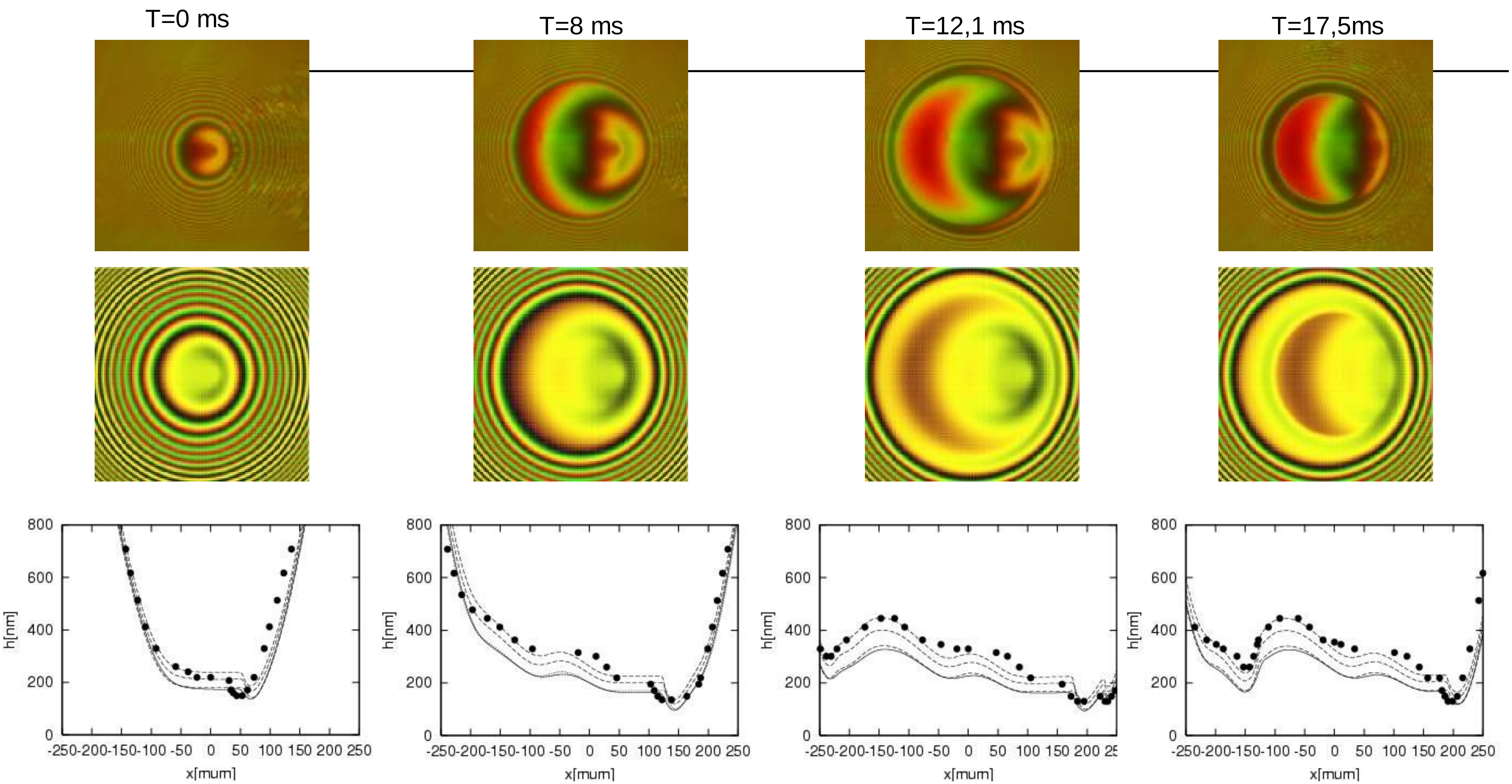




U=1.28 m/s

TIME VARYING: LOAD

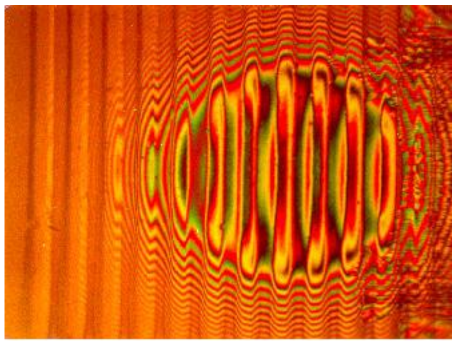
Experimental results: Sakamoto, M., Nishikawa, H., Kaneta, M., Proc. 30th Leeds –Lyon Symposium On Tribology, p391-399 (2004)

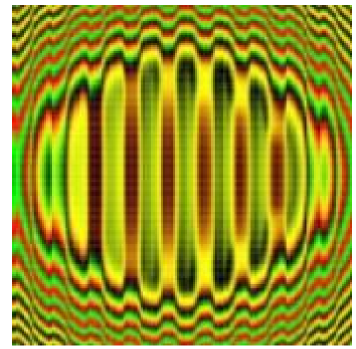


Time Varying: ‘roughness’

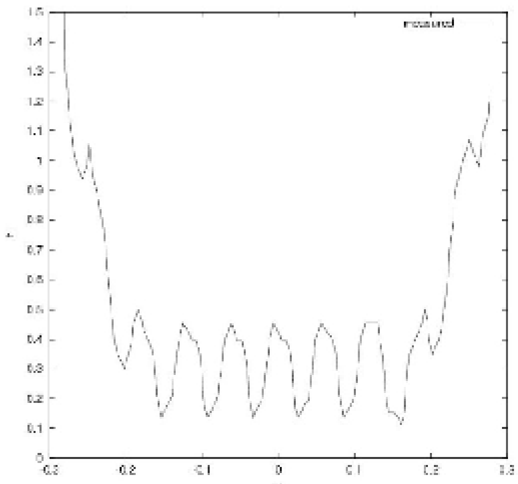
measured

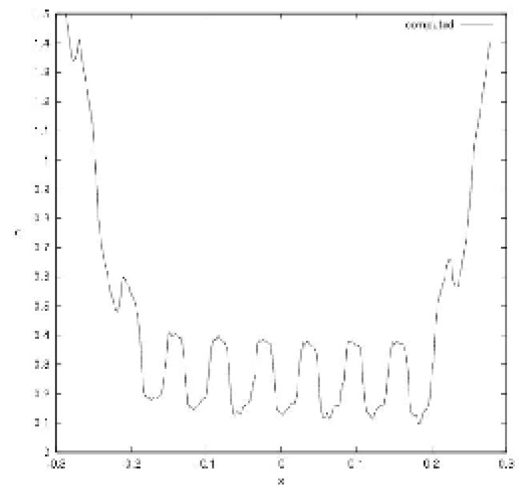
computed





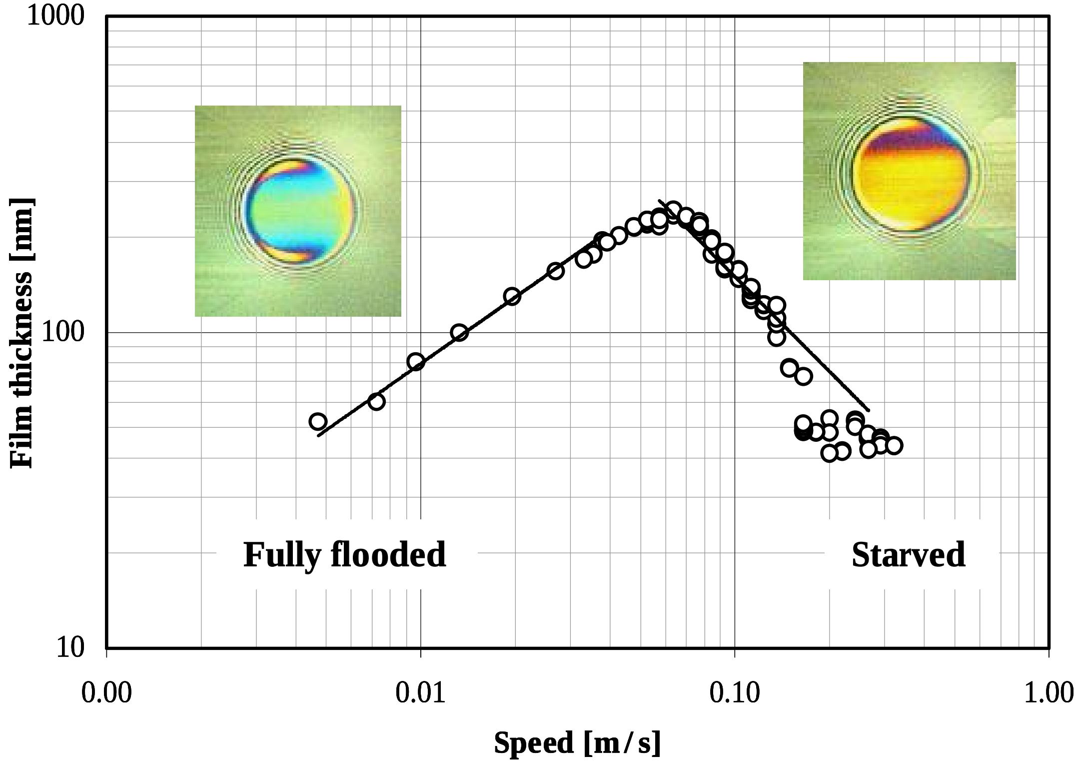
h=280 nm



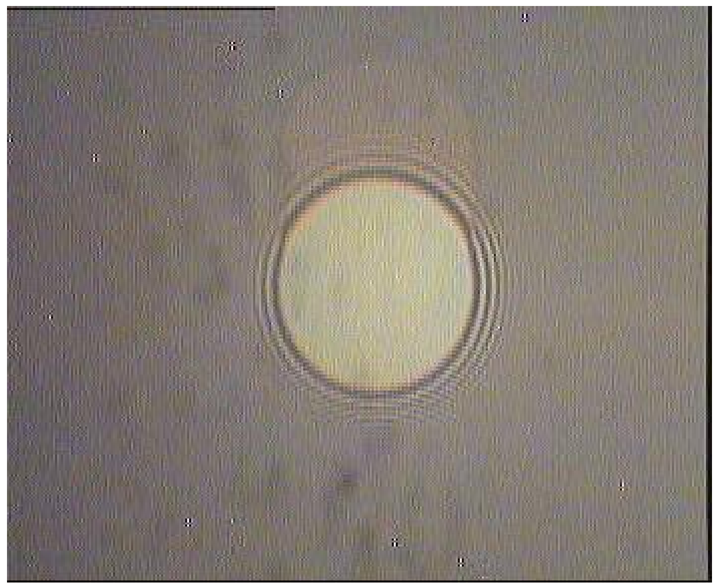


Venner, C.H., Kaneta, M., and Lubrecht, A.A.,
Proceedings 26th Leeds Lyon Symposium on Tribology, p25-36 (2000)

STARVED CONTACTS: EXPERIMENTAL



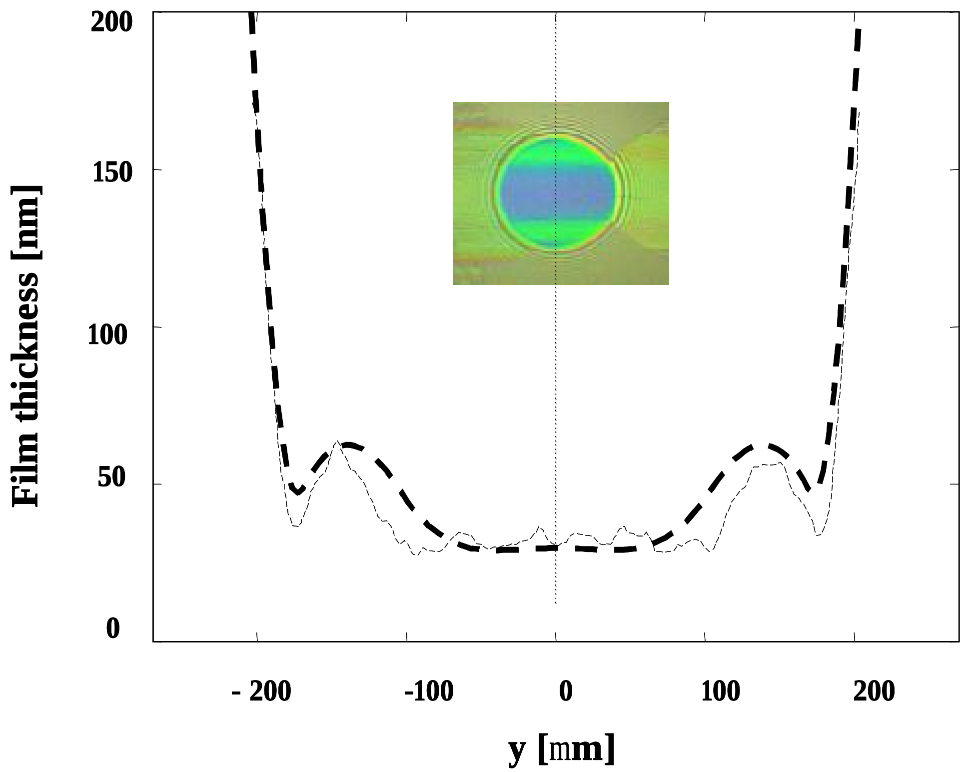
STARVED CONTACTS



STARVED CONTACTS

Direct relation between inlet layer and film thickness in the contact.

Accurate prediction when oil layer thickness correctly modeled.

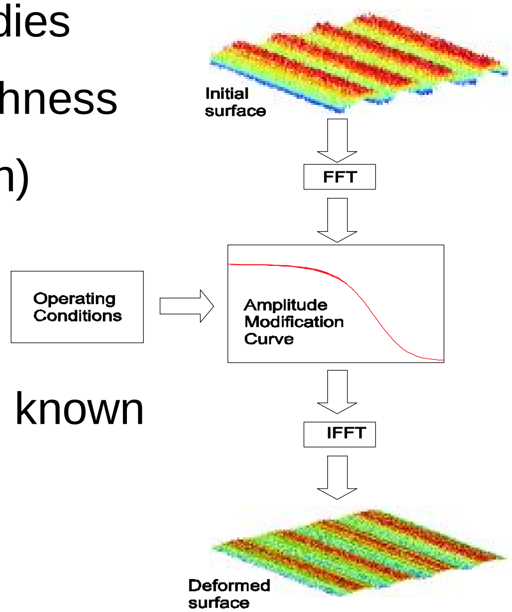


Chevalier, F. Lubrecht, A.A., Cann, P., Dalmaz, G., and Colin, F. Proceedings 22nd Leeds Lyon Symposium on Tribology, p 126-133, (1998)

SINGLE OIL LUBRICATED CONTACT

- Quick numerical solution allowing advanced studies
- Accurate prediction steady state, transient, roughness
- Simple Engineering models (Amplitude reduction)

- Also for starved contacts provided “inlet layer” is known



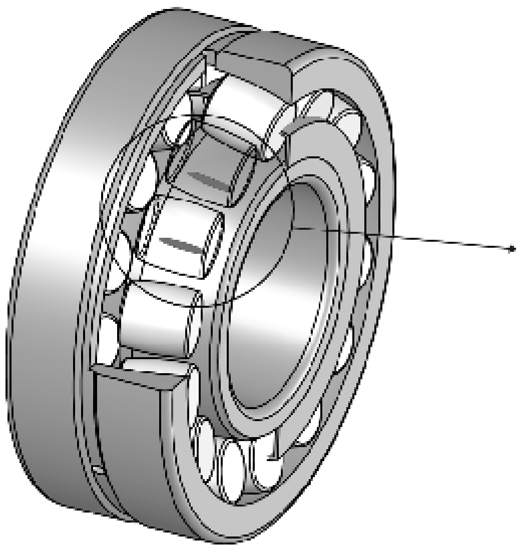
APPLICATION TO REAL BEARINGS ?

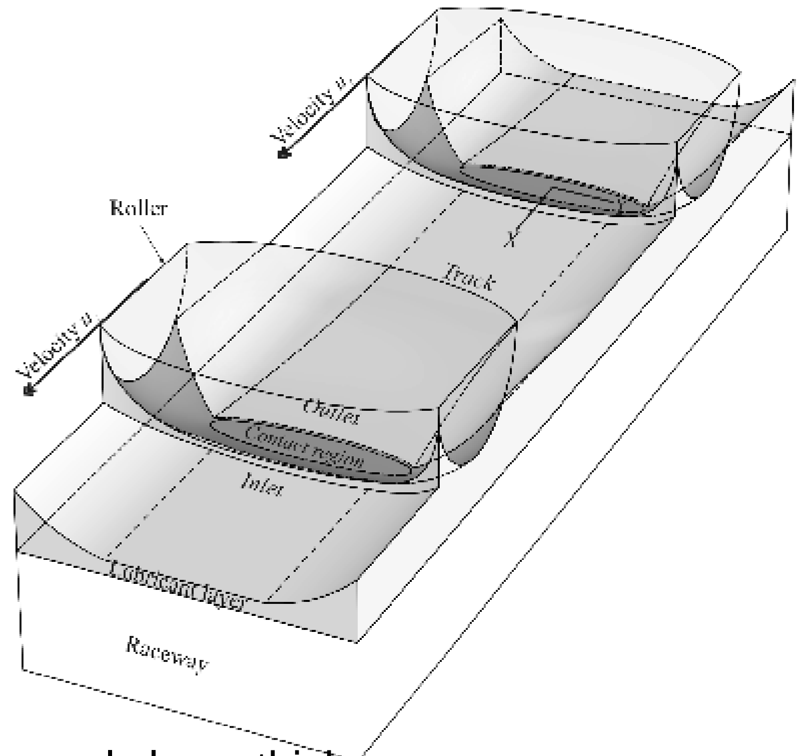
Complications:

- Lubricated with **grease** (model as starved contact)
- Repeated overrolling in very short time
- Billions of overrollings in life-time !!!! (even MG doesn' t help enough)
- Lubricant migration (grease bleeding, cage, centrifugal forces etc.) determines inlet layer of oil on surface to each the contact
- …….

Solution: **Thin Layer flow** model for layer flow, linked to direct relation between layer and film from starved contact.

THIN LAYER FLOW MODEL: INTRO

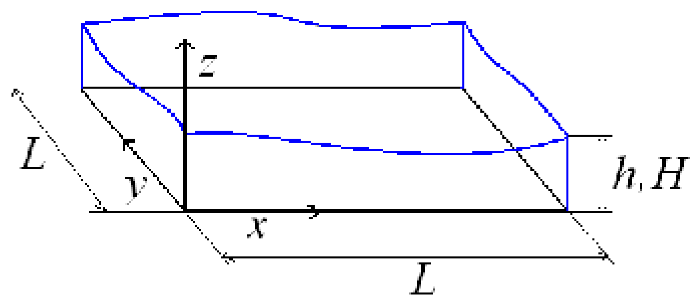




- To develop a model that predicts change supply layer thickness.
- Use model to predict long term film thickness decay.

THIN LAYER FLOW

1. Scale the N-S equations
2. Take the limit as taking the limit of as $\varepsilon \rightarrow 0$
3. Derive equation velocities
4. Insert the velocities into continuity equation.



Navier-Stokes equation (incompressible flow, constant viscosity):

$$r\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)= f_x-\frac{\partial p}{\partial x}+m\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$r\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)= f_y-\frac{\partial p}{\partial y}+m\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$r\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)= f_z-\frac{\partial p}{\partial z}+m\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

THIN LAYER FLOW

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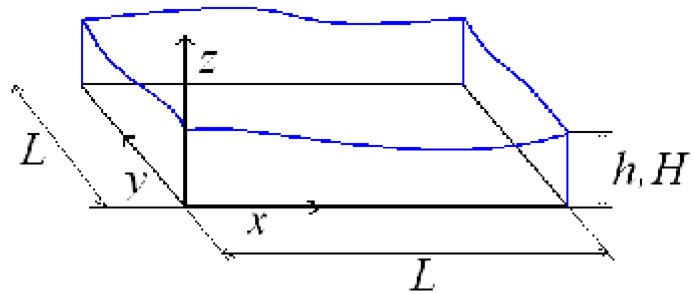

Step 1: $e = \dfrac{H}{L}$ $\qquad W = eU$

$$e^2 \,\mathrm{Re}\left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{u}}{\partial \bar{z}} \right) = \bar{f}_x - \frac{\partial \bar{p}}{\partial \bar{x}} + e^2 \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + e^2 \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2}$$

$$e^2 \,\mathrm{Re}\left(\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{v}}{\partial \bar{z}} \right) = \bar{f}_y - \frac{\partial \bar{p}}{\partial \bar{y}} + e^2 \frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + e^2 \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2}$$

$$e^4 \,\mathrm{Re}\left(\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{w}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{w}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{w}}{\partial \bar{z}} \right) = \bar{f}_z - \frac{\partial \bar{p}}{\partial \bar{z}} + e^4 \frac{\partial^2 \bar{w}}{\partial \bar{x}^2} + e^4 \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + e^2 \frac{\partial^2 \bar{w}}{\partial \bar{z}^2}$$

THIN LAYER FLOW

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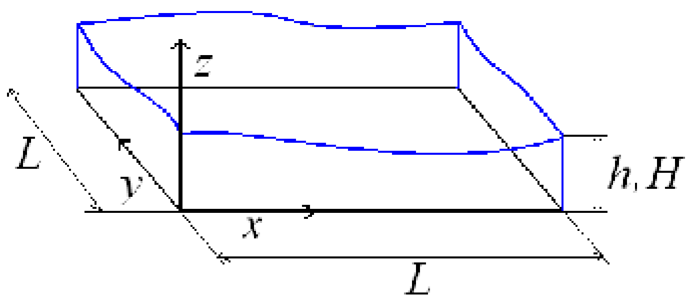

Step 2: $e = \frac{H}{L} \quad W = eU$

$$e^2 \mathrm{Re}\left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{u}}{\partial \bar{z}}\right) = \bar{f}_x - \frac{\partial \bar{p}}{\partial \bar{x}} + e^2\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + e^2\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2}$$

$$e^2 \mathrm{Re}\left(\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{v}}{\partial \bar{z}}\right) = \bar{f}_y - \frac{\partial \bar{p}}{\partial \bar{y}} + e^2\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + e^2\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2}$$

$$e^4 \mathrm{Re}\left(\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{w}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{w}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{w}}{\partial \bar{z}}\right) = \bar{f}_z - \frac{\partial \bar{p}}{\partial \bar{z}} + e^4\frac{\partial^2 \bar{w}}{\partial \bar{x}^2} + e^4\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + e^2\frac{\partial^2 \bar{w}}{\partial \bar{z}^2}$$

THIN LAYER FLOW

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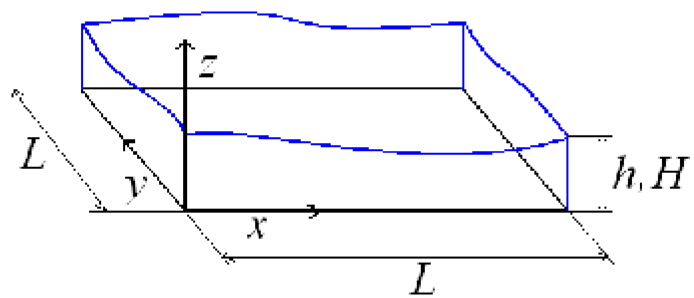

Step 2:

$$0 = f_x - \frac{\partial p}{\partial x} + m\left(\frac{\partial^2 u}{\partial z^2}\right)$$

$$0 = f_y - \frac{\partial p}{\partial y} + m\left(\frac{\partial^2 v}{\partial z^2}\right)$$

$$0 = f_z - \frac{\partial p}{\partial z}$$

THIN LAYER FLOW

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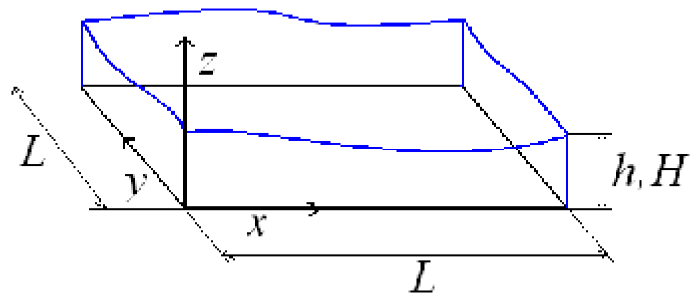

Step 2:

$$0 = f_x - \frac{\partial p}{\partial x} + m\left(\frac{\partial^2 u}{\partial z^2}\right)$$

$$0 = f_y - \frac{\partial p}{\partial y} + m\left(\frac{\partial^2 v}{\partial z^2}\right)$$

$$0 = f_z - \frac{\partial p}{\partial z}$$

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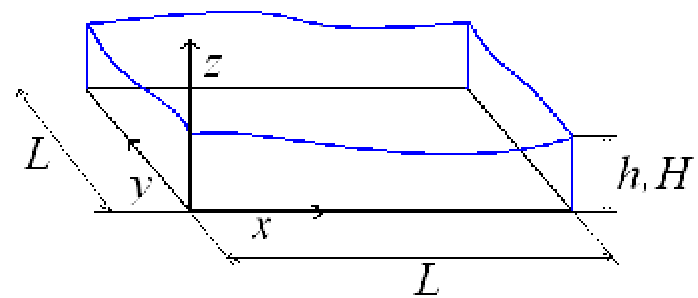

Step 3:

$$p = f_z (z - h) - t_c k + p_0$$

$$\langle u \rangle = \frac{1}{h}\int_0^h u\,dz = \frac{h^2}{3m}\left[f_x + \tfrac{3}{8} h \frac{\partial f_z}{\partial x} + f_z \frac{\partial h}{\partial x} + t_s \left(\frac{\partial^3 h}{\partial x^3} + \frac{\partial^3 h}{\partial y^2 \partial x} \right) \right]$$

$$\langle v \rangle = \frac{1}{h}\int_0^h v\,dz = \frac{h^2}{3m}\left[f_y + \tfrac{3}{8} h \frac{\partial f_z}{\partial y} + f_z \frac{\partial h}{\partial y} + t_s \left(\frac{\partial^3 h}{\partial x^2 \partial y} + \frac{\partial^3 h}{\partial y^3} \right) \right]$$

THIN LAYER APPROXIMATION

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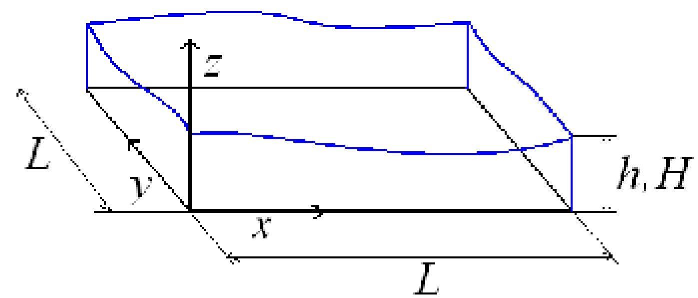

Step 4:

$$\frac{1}{3m}\frac{\partial}{\partial x}\left(h^3\left[f_x + \tfrac{3}{8}h\frac{\partial f_z}{\partial x} + f_z\frac{\partial h}{\partial x} + t_s\left(\frac{\partial^3 h}{\partial x^3} + \frac{\partial^3 h}{\partial y^2 \partial x}\right)\right]\right) + \ldots$$

$$\frac{1}{3m}\frac{\partial}{\partial y}\left(h^3\left[f_y + \tfrac{3}{8}h\frac{\partial f_z}{\partial y} + f_z\frac{\partial h}{\partial y} + t_s\left(\frac{\partial^3 h}{\partial x^2 \partial y} + \frac{\partial^3 h}{\partial y^3}\right)\right]\right) + \frac{\partial h}{\partial t} = 0$$

van Zoelen, M. T.; Venner, C. H. & Lugt, P. M. "*Free Surface Thin Layer Flow on Bearing Raceways,*" *Journal of Tribology, ASME,* **2008**, *130*, 021802

THIN LAYER APPROXIMATION

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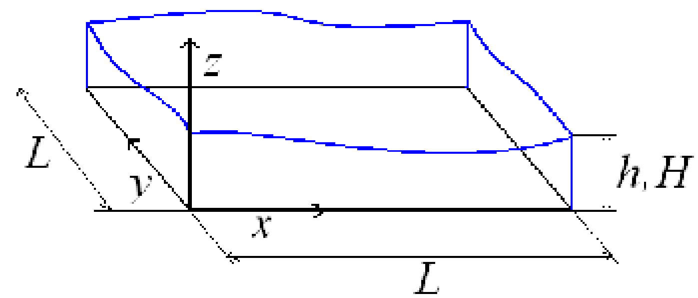


Step 4:

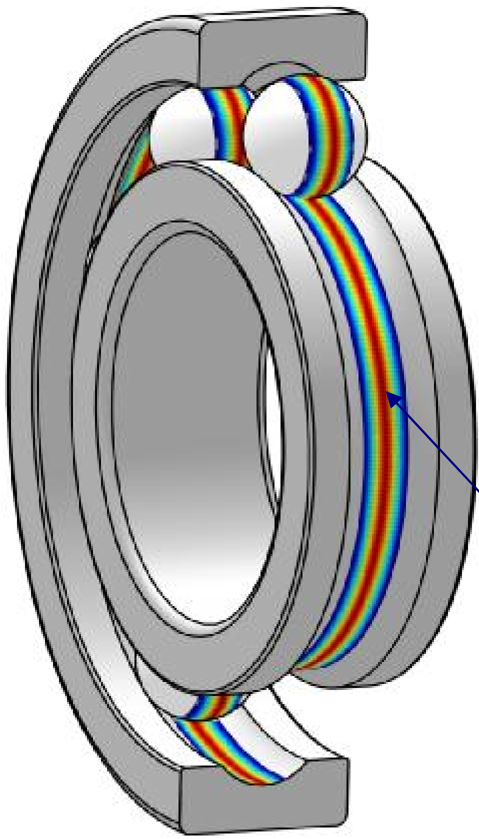
$$\frac{1}{3m}\frac{\partial}{\partial x}\left(h^3\left[f_x + \tfrac{3}{8}h\frac{\partial f_z}{\partial x} + f_z\frac{\partial h}{\partial x} + t_s\left(\frac{\partial^3 h}{\partial x^3} + \frac{\partial^3 h}{\partial y^2 \partial x}\right)\right]\right) + \ldots$$

$$\frac{1}{3m}\frac{\partial}{\partial y}\left(h^3\left[f_y + \tfrac{3}{8}h\frac{\partial f_z}{\partial y} + f_z\frac{\partial h}{\partial y} + t_s\left(\frac{\partial^3 h}{\partial x^2 \partial y} + \frac{\partial^3 h}{\partial y^3}\right)\right]\right) + \frac{\partial h}{\partial t} = 0$$

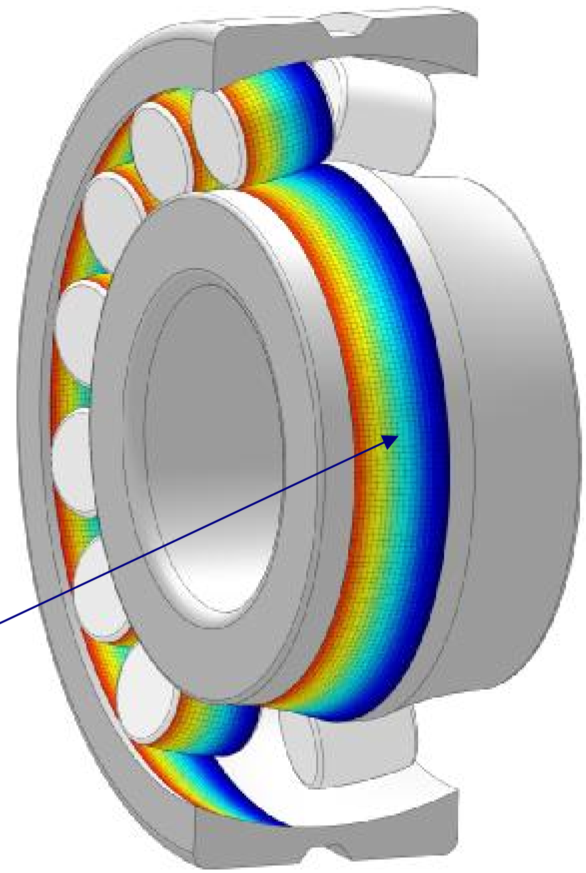
van Zoelen, M. T.; Venner, C. H. & Lugt, P. M. "*Free Surface Thin Layer Flow on Bearing Raceways,*" *Journal of Tribology, ASME,* **2008**, *130*, 021802

THIN LAYER FLOW IN BEARINGS

Contact pressure effect



Centrifugal effect



Lubricant film

thickness distribution

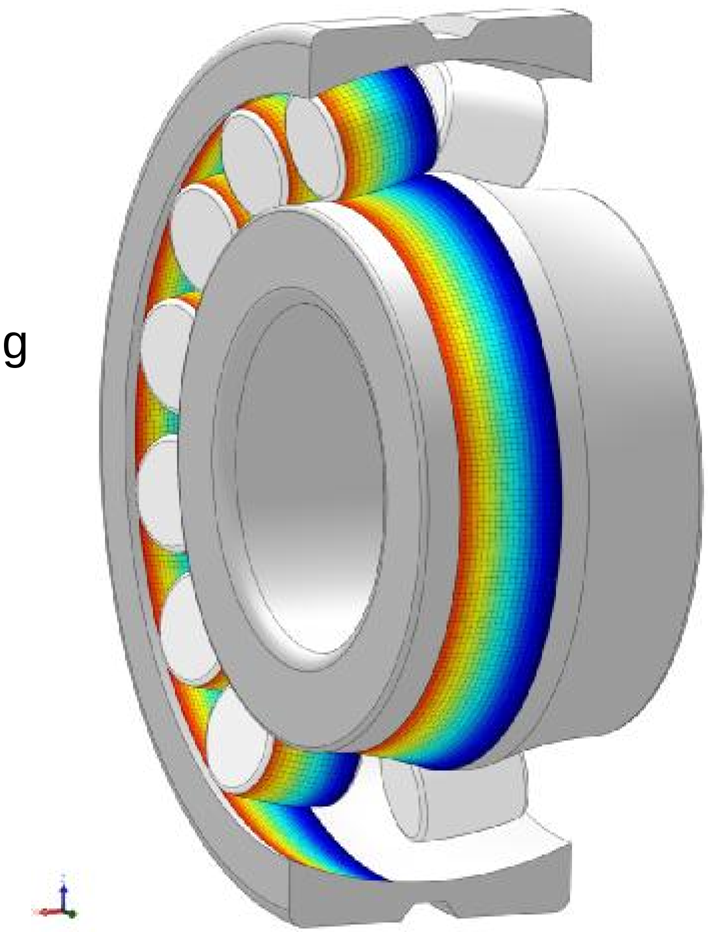
SIMPLIFICATION

2D à 1D:

- Equipartition
- Contact pressure smoothening
- Surface tension

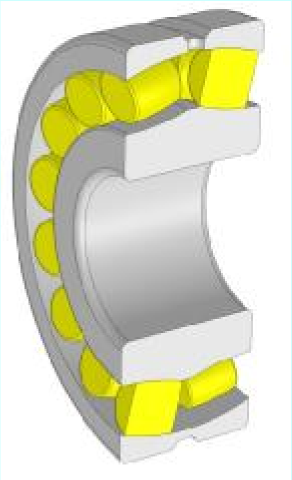
$$\frac{1}{3m}\frac{\partial}{\partial y}\left(h^3 f_x\right)+\frac{\partial h}{\partial t}=0$$

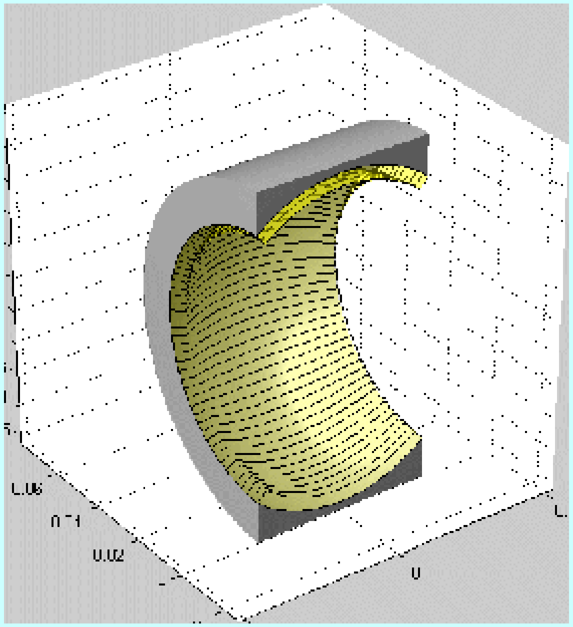
- Hyperbolic equation, easily solved by method of characteristics



CENTRIFUGAL EFFECTS RACEWAY

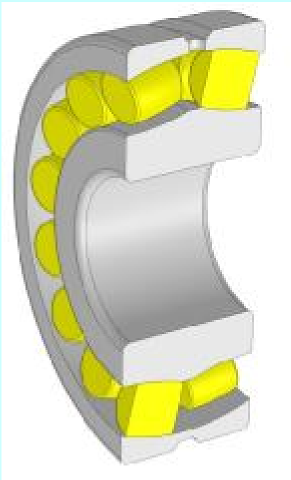
Example

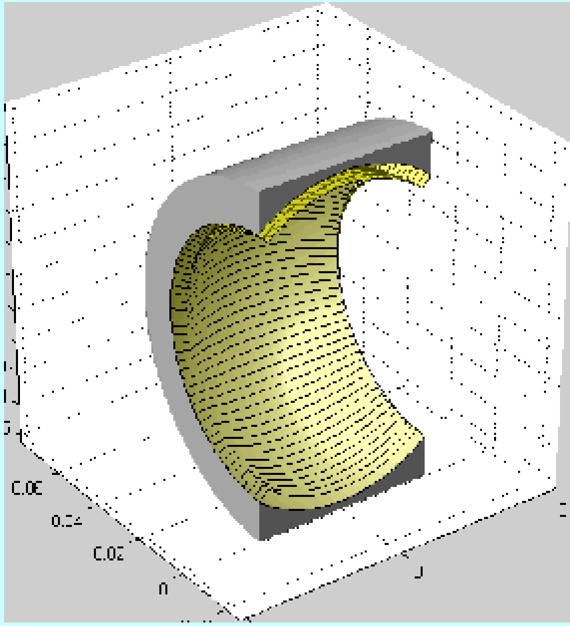




CENTRIFUGAL EFFECTS RACEWAY

Example





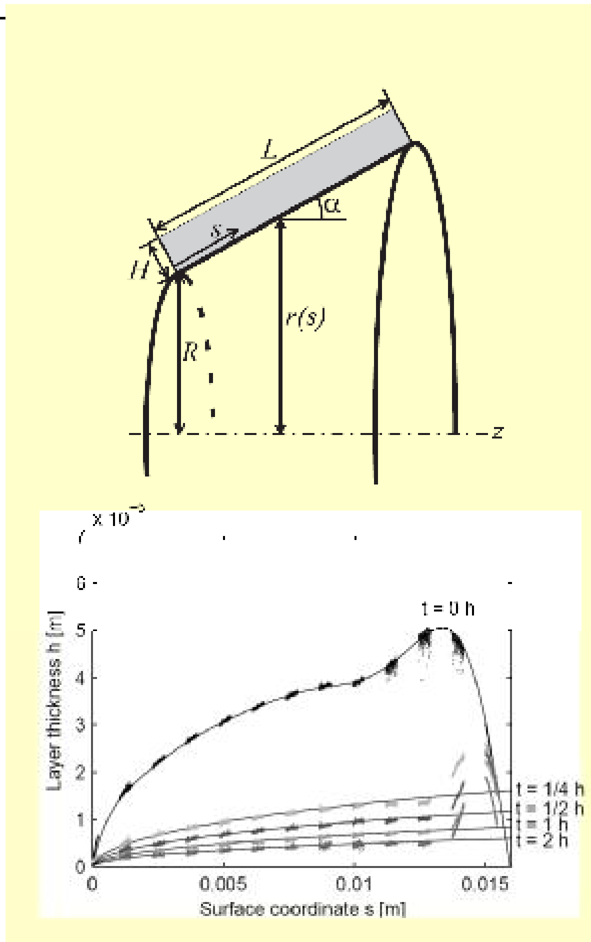
CENTRIFUGAL EFFECT RACEWAY: VALIDATION

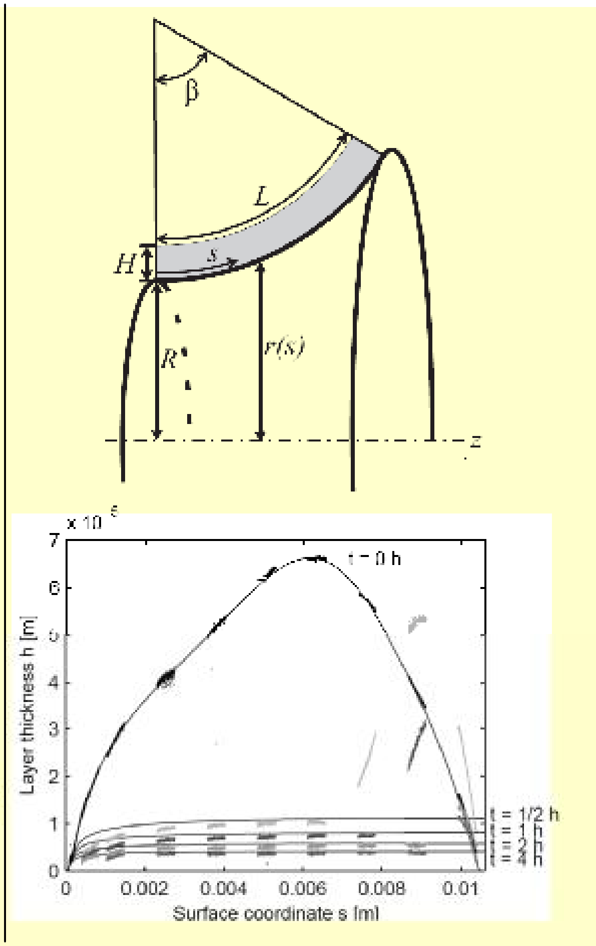
Flow equation

$$\frac{1}{r}\frac{\partial}{\partial s}\left(\frac{h^3}{3h_0} r\ f_s\right)+\frac{\partial h}{\partial t}=0$$

Body force equation

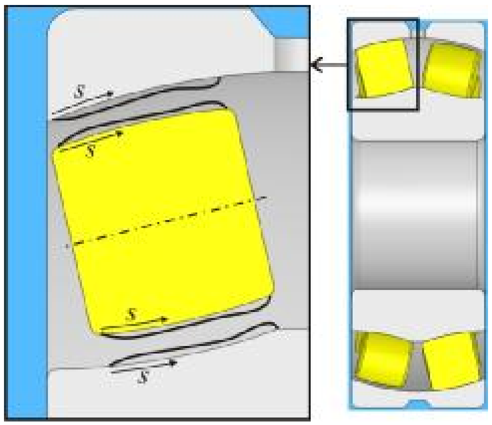
$$f_s = r\Omega^2 r\frac{dr}{ds}$$

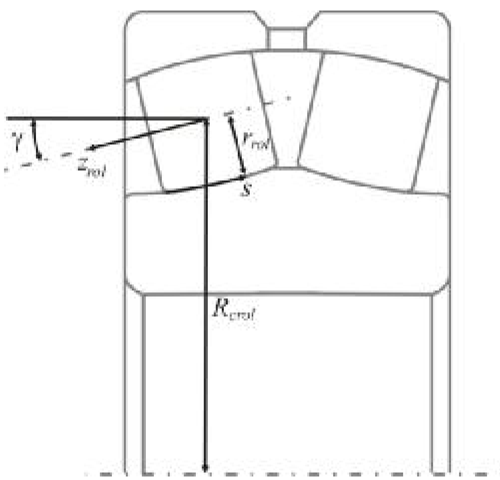




van Zoelen, M. T.; Venner, C. H. & Lugt, P. M. "*Free Surface Thin Layer Flow on Bearing Raceways,*" *Journal of Tribology, ASME,* **2008**, *130*, 021802

CENTRIFUGAL EFFECT ROLLER





Flow equation

$$\frac{1}{r}\frac{\partial}{\partial s}\left(\frac{h^3}{3h_0} r\ f_s\right)+\frac{\partial h}{\partial t}=0$$

Body force equation

Raceways:

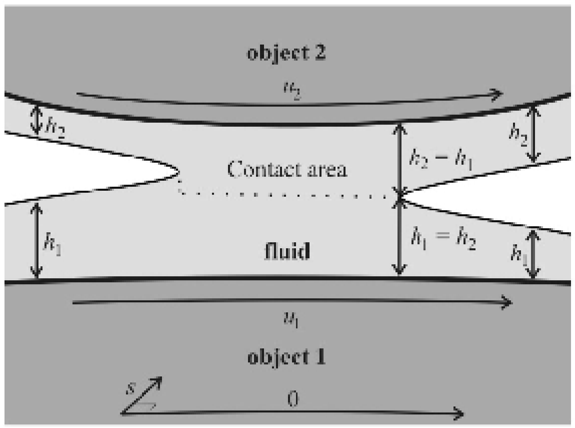
$$f_{s,rw} = r\Omega_{rw}{}^2 r\,\frac{dr}{ds}$$

Rollers:

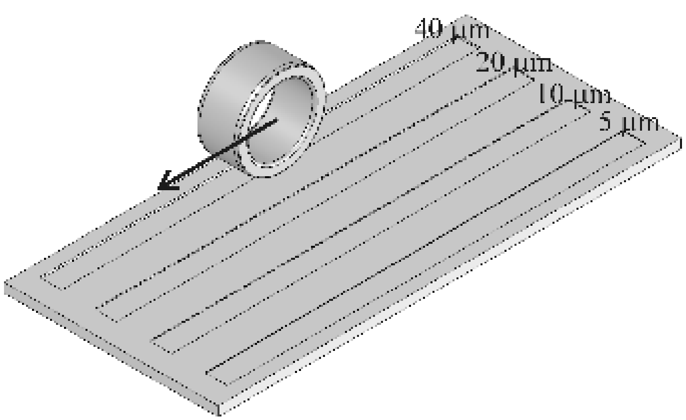
$$f_{s,rol} = r\ \Omega_{ca}{}^2\left(\sin^2(g)\,z_{rol}+\sin(g)\,R_{crol}\right)\frac{dz_{rol}}{ds}$$
$$+\left(\left(\tfrac{1}{2}\cos^2(g)+\tfrac{1}{2}\right)\Omega_{ca}{}^2+2\Omega_{ca}\Omega_{rol}\cos(g)+\Omega_{rol}{}^2\right) r\,r_{rol}\,\frac{dr_{rol}}{ds}$$

COMBINING LAYERS: EQUIPARTION

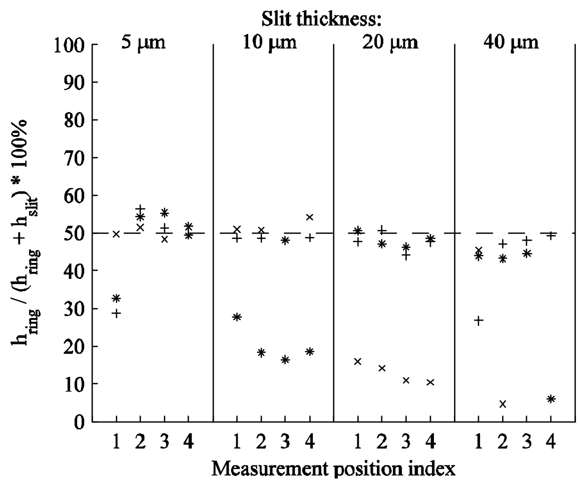
The concept



Measurement setup



Measurement Results

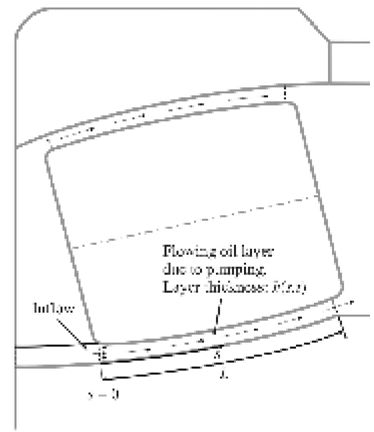


Measurements have been carried out by H. de Ruig and R. Meeuwenoord at SKF ERC

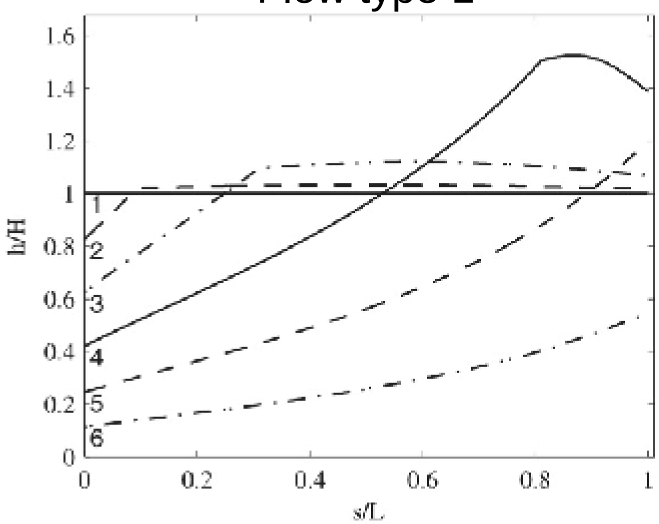
CENTRIFUGAL EFFECT: BEARING

- Time steps:

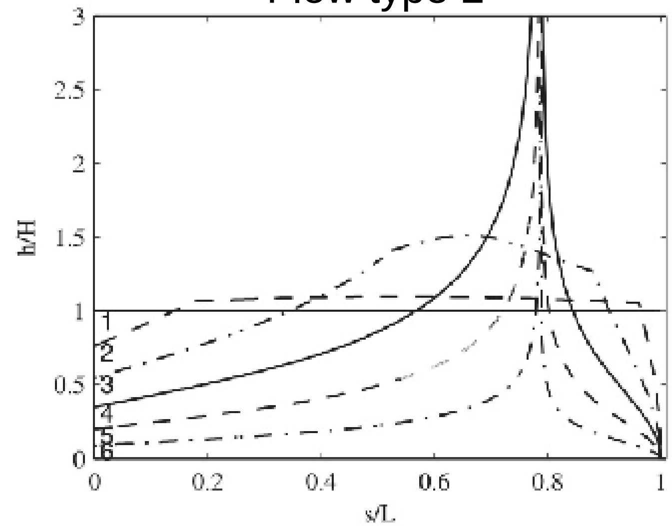
$$t/t = \frac{h}{H^2 r \, \Omega^2} = 0,\ 0.3,\ 1,\ 3,\ 10,\ 50$$



Flow type 1



Flow type 2



CONTACT PRESSURE: BEARING

Mass conservation

$$\frac{\partial \tilde{h}_\infty}{\partial t} = -\frac{1}{r_0 l_t}\frac{\partial \hat{q}_y}{\partial y}$$

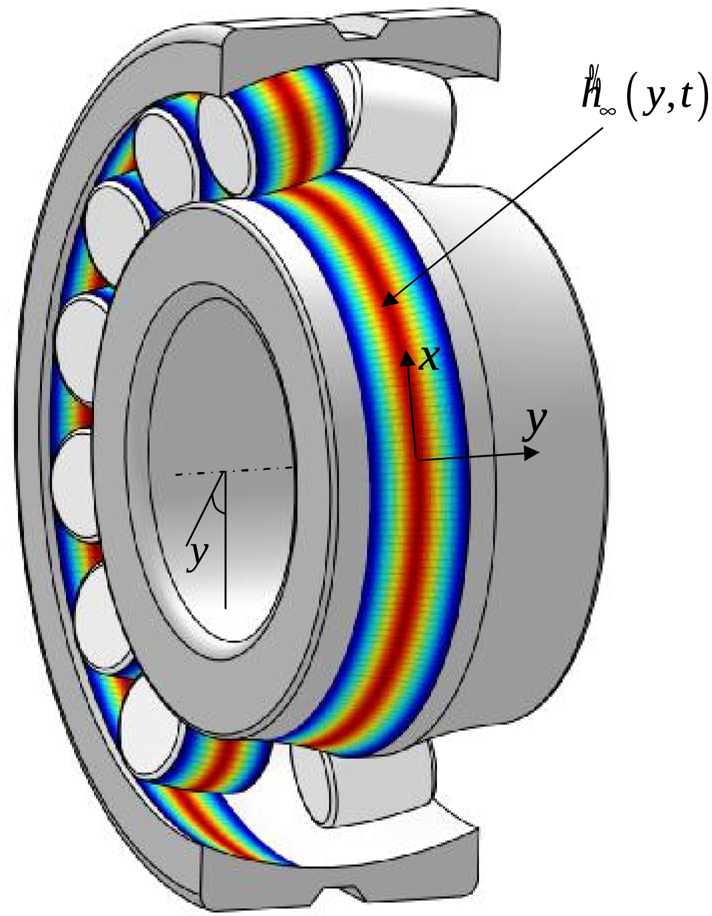
Mass flow in EHL contacts

$$\hat{q}_y\left(y,t\right) = \sum_{k=1}^{n_c} \hat{q}_{y,k}$$

$$\hat{q}_{y,k}\left(y,t\right) = \frac{1}{2p}\int_0^{2p}\int_{a^-}^{a^+}\left(-\frac{r h^3}{12h}\frac{\partial p}{\partial y}\right)_k dx\,dy$$

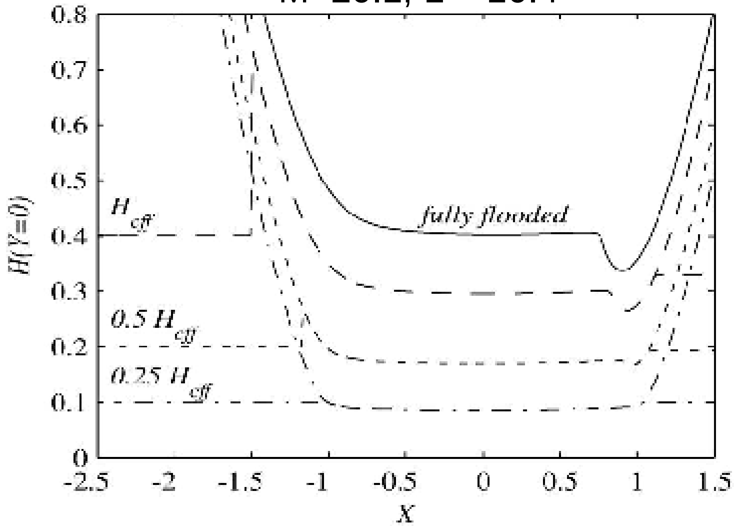
$$h = h\left(p\right) \qquad p = p\left(x,y,y\,,t\right)$$

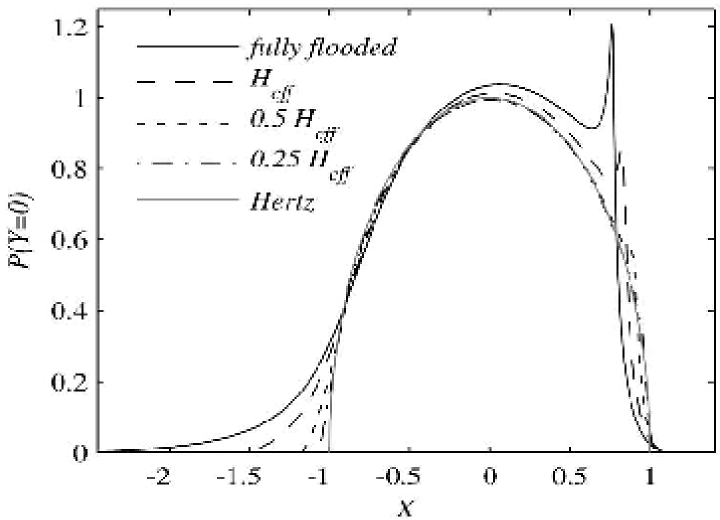
$$r = r\left(p\right) \qquad h = h\left(x,y,y\,,t\right)$$



CONNECTION TO "INSIDE CONTACT"

M=20.1, L = 10.4





Layer thickness

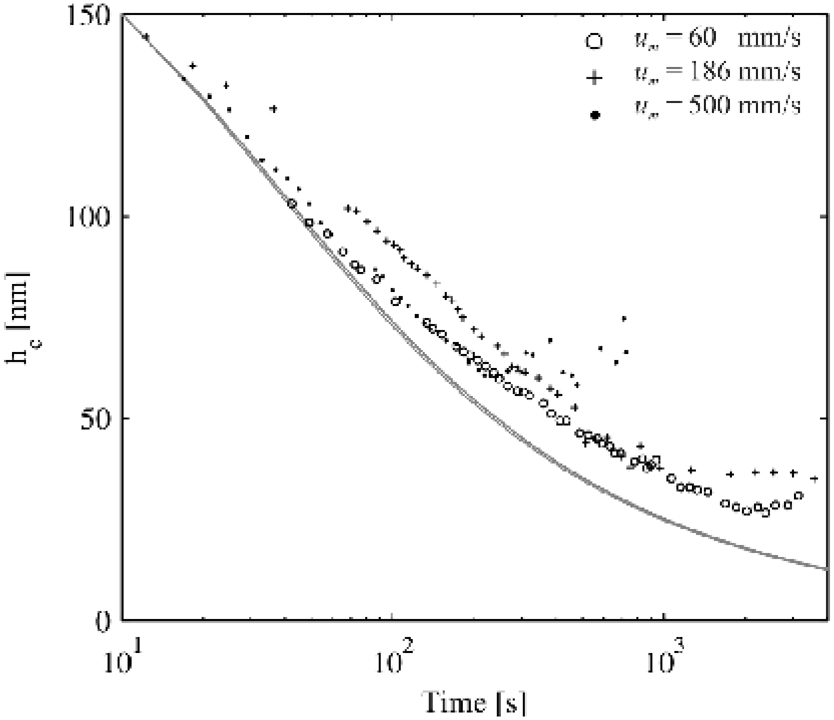
$$\lim_{h_{oil}\to 0} h = \frac{2h_\infty}{\bar{r}}$$

Pressure

$$\lim_{h_{oil}\to 0} p = p_h\sqrt{1-\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2}$$

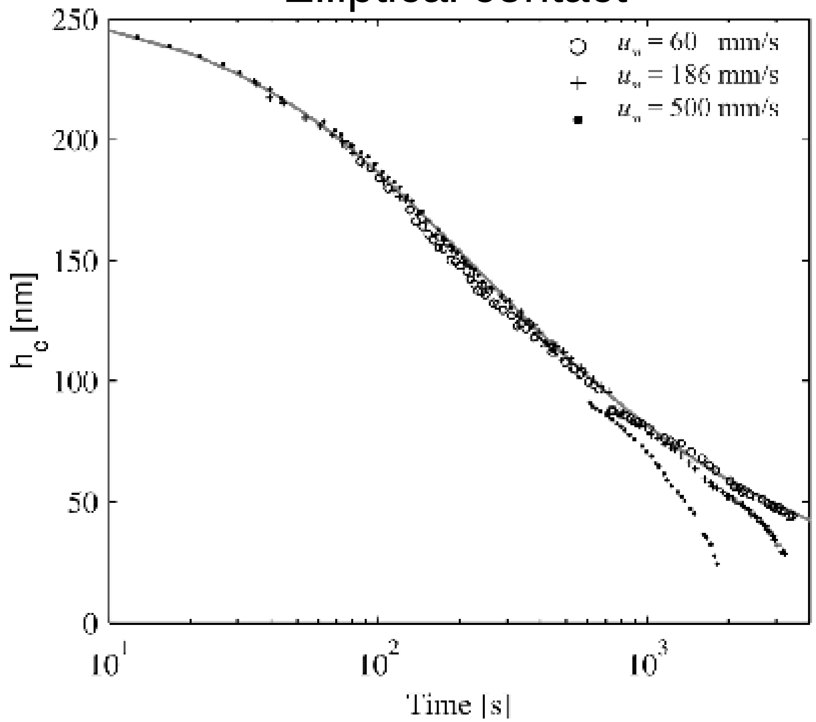
SINGLE CONTACT: VALIDATION

Circular contact



F = 20 N, p_h = 0.5 GPa, $\eta_0 \approx 0.8$ Pa.s

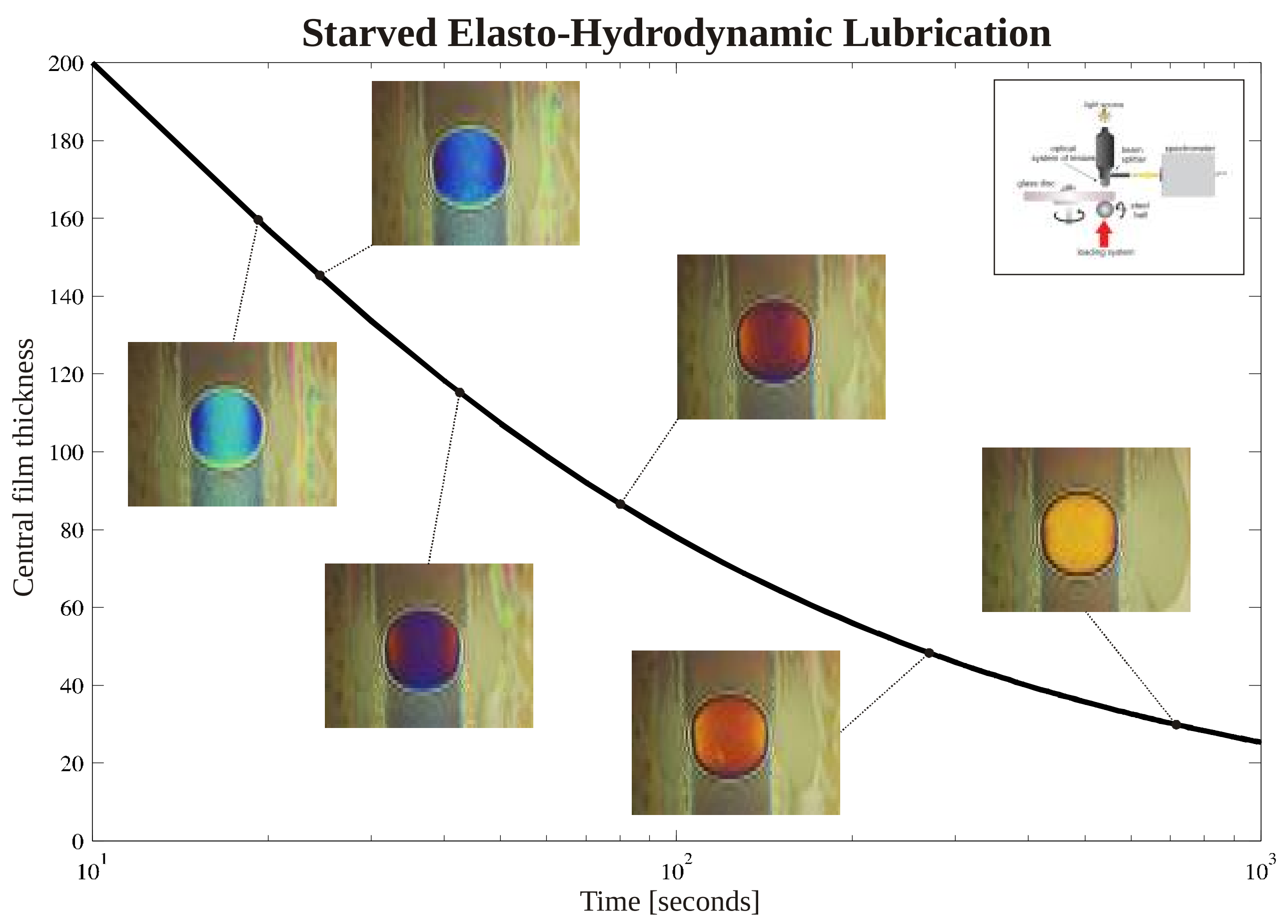
Elliptical contact



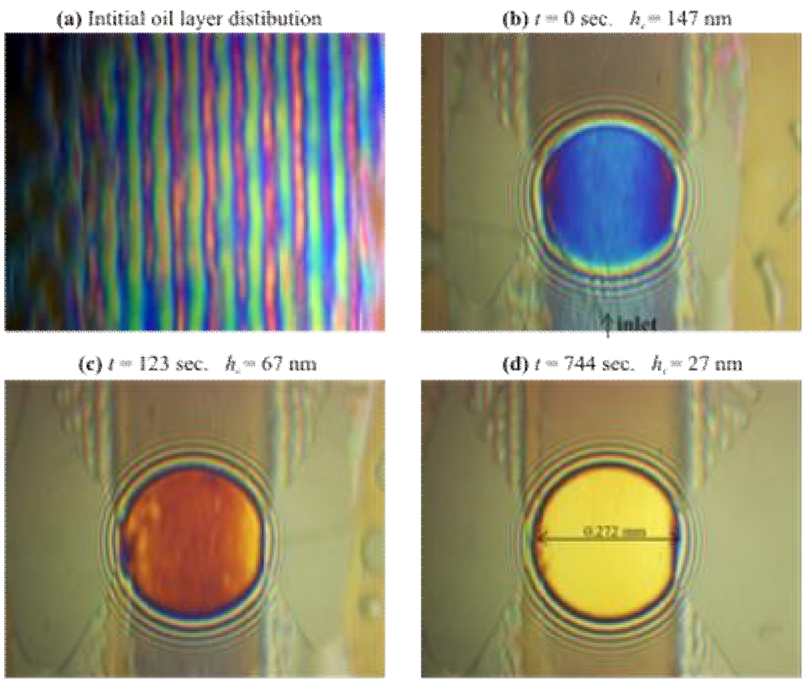
F = 30 N, p_h = 0.33 GPa, $\eta_0 \approx 0.85$ Pa.s

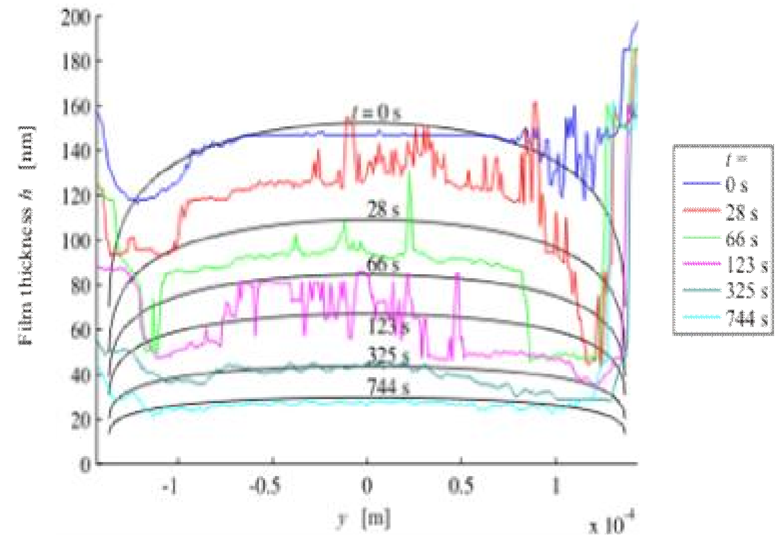
van Zoelen, M. T.; Venner, C. H. & Lugt, P. M. "Prediction of Film Thickness Decay in Starved EHL Contacts using a Thin Layer Flow Model," *Journal of Engineering Tribology, ImechE*, **2009**, *223*, In Press.

SINGLE CONTACT: VALIDATION

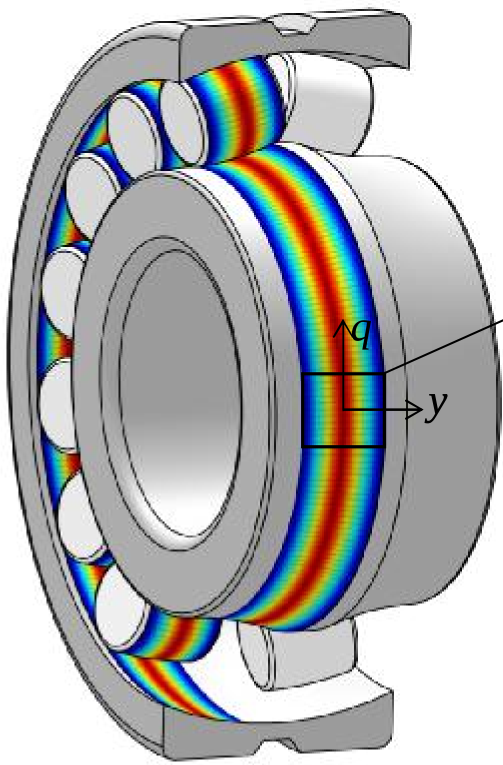


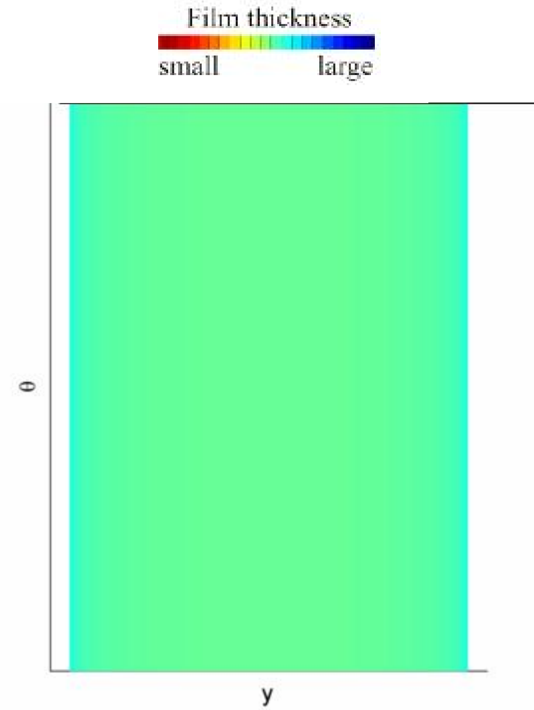
SINGLE CONTACT: VALIDATION

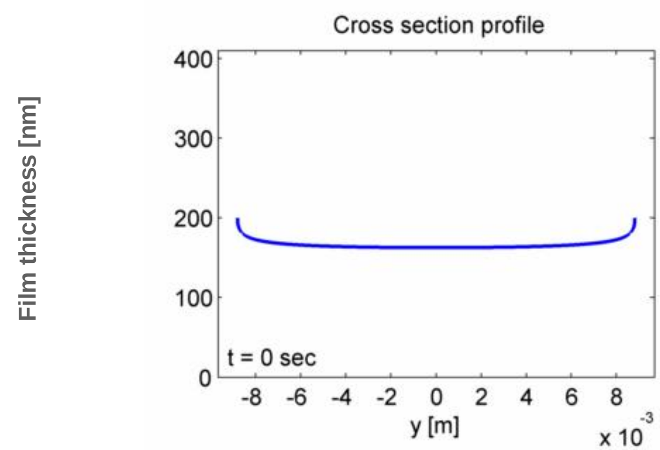




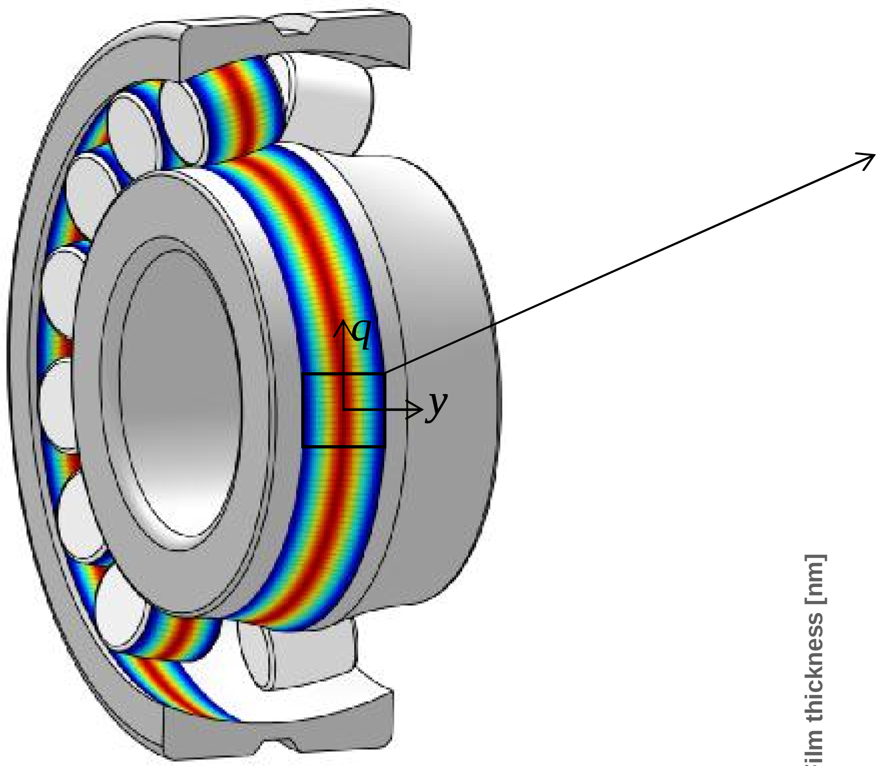
CONTACT PRESSURE: BEARING

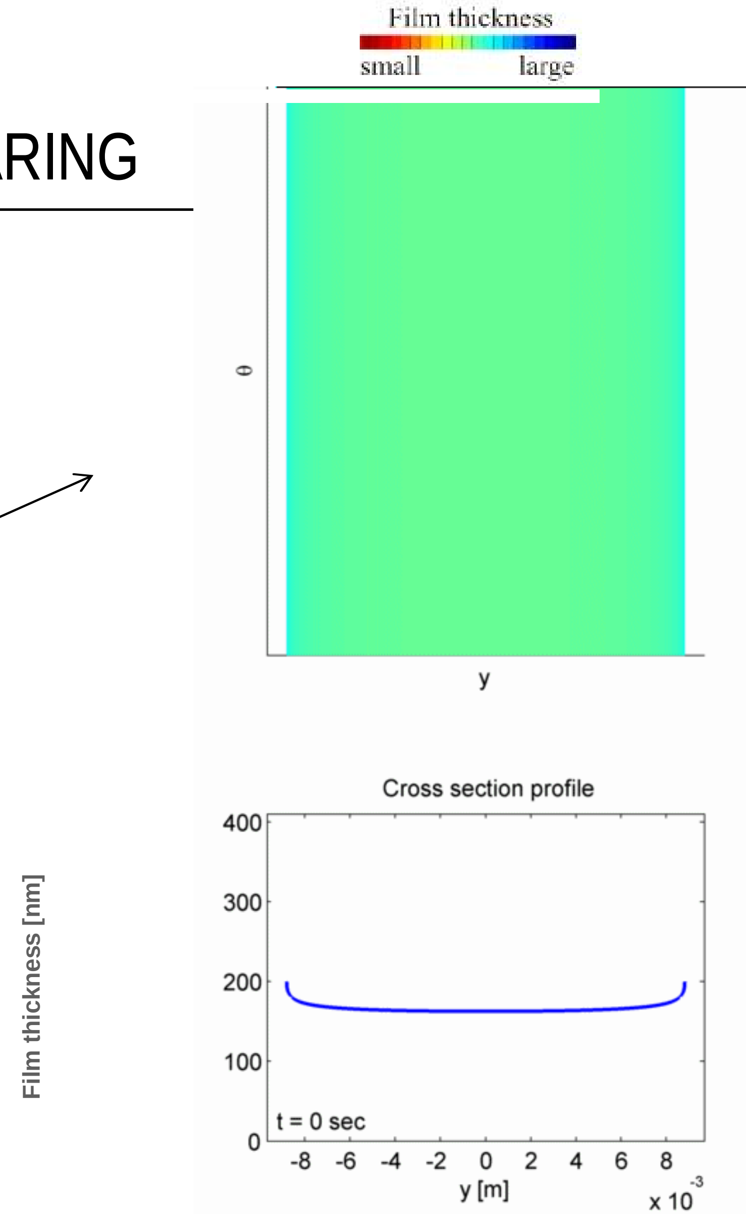




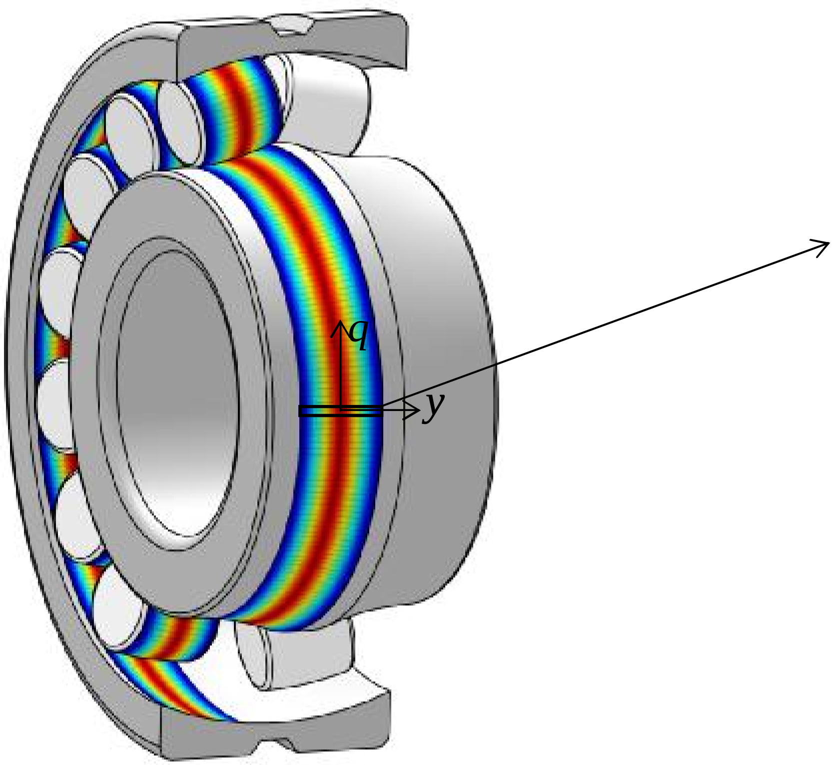


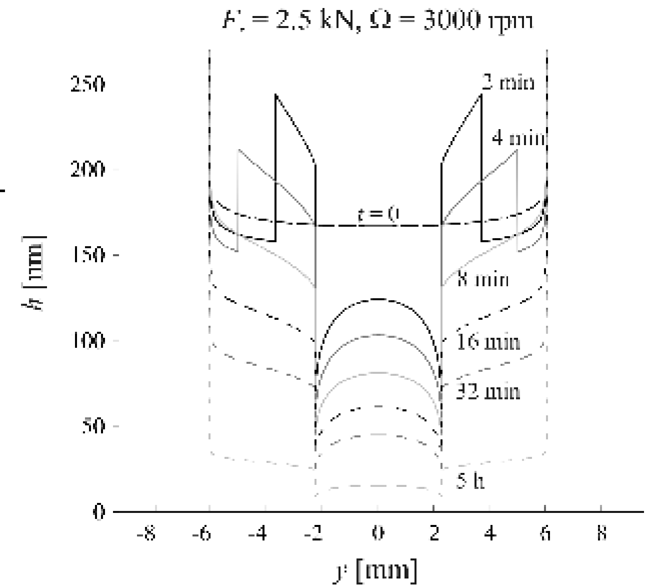
CONTACT PRESSURE: BEARING

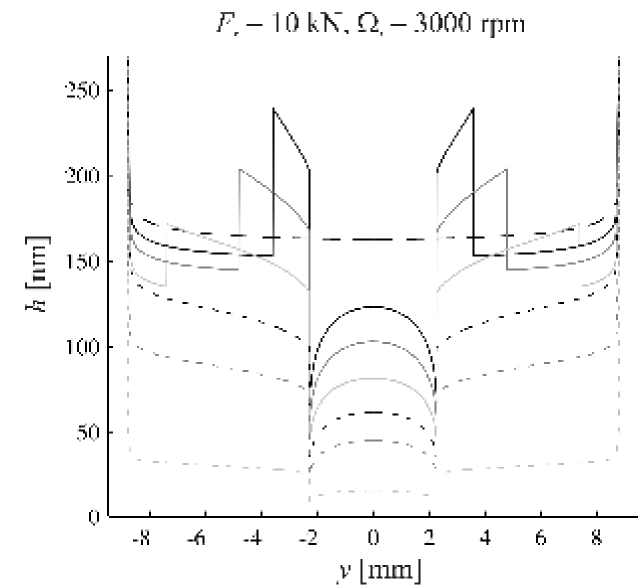




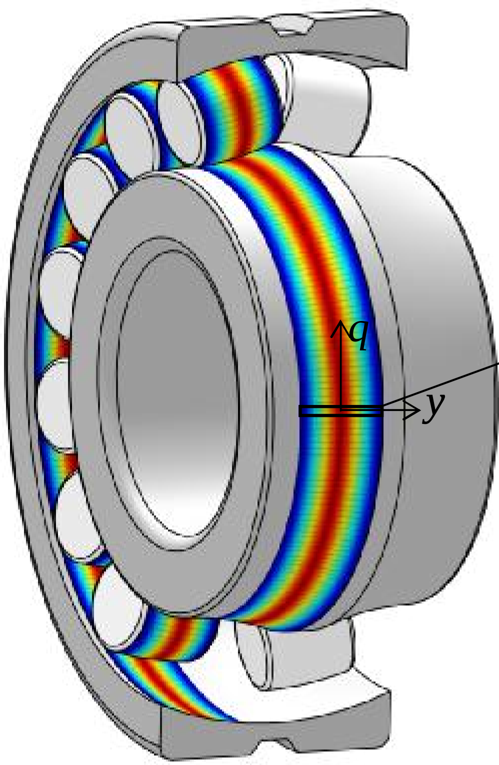
VARYING BEARING LOAD

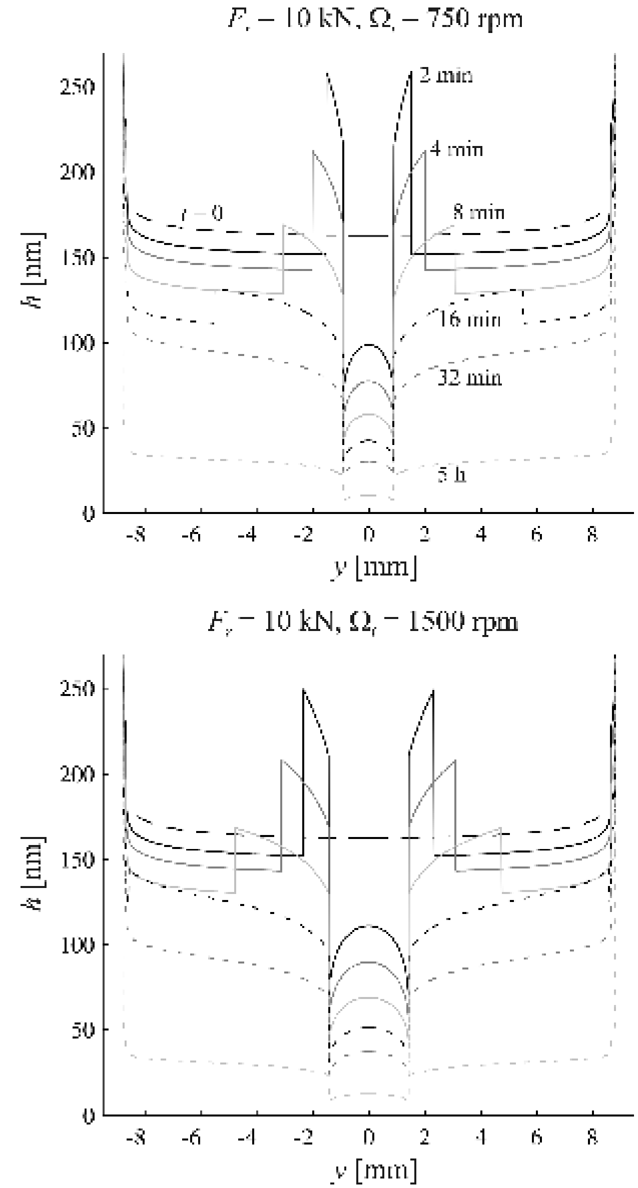




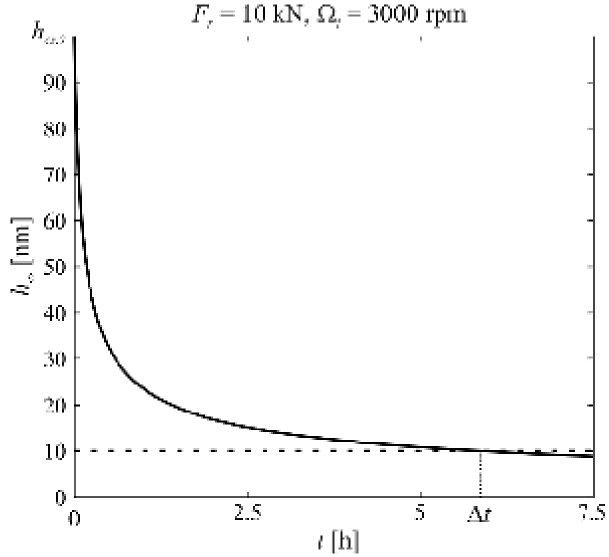


VARYING BEARING SPEED

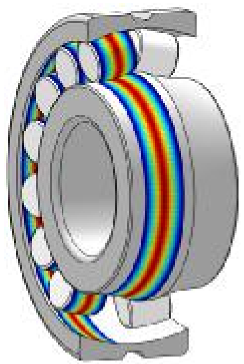




FILM THICKNESS DECAY

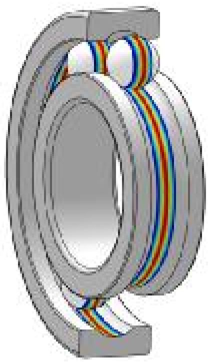


Spherical Roller Bearing



Load [kN]	Speed [rpm]	Δt [h]
10	*750*	**2.5**
10	*1500*	**3.8**
10	*3000*	**5.8**
5	*3000*	**5.7**
2.5	*3000*	**5.6**

Deep groove Ball Bearing



Load [kN]	Speed [rpm]	Δt [h]
10	*750*	**0.048**
10	*1500*	**0.072**
10	*3000*	**0.109**
5	*3000*	**0.105**
2.5	*3000*	**0.100**

CONCLUSION

- Film decay model for bearings developed from based on:

  Thin layer flow model

  Starved EHL

- Model is developed to predict change of supply layer.
  - Centrifugal effects
  - Contact pressure effects

- Model is validated experimentally.

CONCLUSION

- Larger layer thickness decay for ball bearing
- Decay depends on speed.
- Decay depends weekly on the load
- Decay periods are short **à** significant lubricant supply to the track.

CONCLUSION

- Include lubricant supply mechanisms.
- Layer smoothening.
- Comparison with bearing tests: qualitative/quantitative

- ……AND MANY OTHER INTERESTING THINGS…..