THIN LAYER FLOW MODELING
PREDICTION OF FILM DECAY IN EHL CONTACTS AND ROLLING ELEMENT BEARINGS

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ROLLING ELEMENT BEARINGS
SINGLE CONTACT MODELLING (EHL)

Experimental

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SINGLE CONTACT MODELING (EHL)

Flow: Navier Stokes, Narrow Gap assumption:

\[
\frac{\partial}{\partial X} \left( \varepsilon \frac{\partial P}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \frac{\partial P}{\partial Y} \right) - \Lambda(T) \frac{\partial (\theta \, \bar{p} \, H)}{\partial X} - \frac{\partial (\theta \, \bar{p} \, H)}{\partial T} = 0
\]

Gap height \( h \): undeformed shape + elastic deformation

\[
H(X, Y, T) = -\Delta(T) + \frac{X^2}{2} + \frac{Y^2}{2} + \frac{2}{\pi^2} \int_S \frac{P(X', Y', T) \, dX' \, dY'}{\sqrt{(X - X')^2 + (Y - Y')^2}}
\]

Equation of Motion

\[
\frac{1}{\Omega^2} \frac{d^2 \Delta}{dT^2} + \frac{3}{2\pi} \int_S P(X, Y, T) \, dXdY + \overline{K} \cdot \Delta = 1 + \overline{K} \Delta_\infty
\]

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Conceptual approach:

- Identify **problematic** components responsible for computational slowness (slow convergence, multi-summations).
- Design **accurate** representation for **efficient** solution (computation).

**Appearances:**

- Standard: **Geometric Multigrid**
- Advanced: General Systems: **AMG**
- Advanced: Physics, Chemistry, Particles, etc.
GEOMETRIC MULTIGRID

- Iterative Process **bad solver** but **good smoother**
- Smooth error can accurately be approximated on **coarser grid**
- Solve error on coarser grid
- Correct fine grid solution
- Result: **grid independent** high convergence rate \(O(0.1)\), work \(O(N)\)

- **Geometric MG**: Fix coarsening and intergrid operators, design good smoother. **Advantage**: Principle straightforward, non linearity equally efficient. **Disadvantage**: Sometimes not trivial (stability for integral equations)

- **EHL**: Elastic deformation integrals (Multilevel Multi-Integration)

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ALGEBRAIC MULTIGRID (AMG)

System of equations

\[ Au = f \]

- **Fix** iterative scheme (GS relaxation, Kaczmarz relaxation)
- Matrix \( A \) and iterative scheme determine coarsening and intergrid operators such that slow to converge error is accurately approximated.

- **Advantage**: Little knowledge of system required, Very robust!
- **Disadvantage**: Set up more expensive (only once), Non-linearity more involved (but still no global linearization needed)
AMG: Example

\[
\frac{\partial^2 u}{\partial x^2} + \varepsilon \frac{\partial^2 u}{\partial y^2} = f(x, y)
\]
AMG: Example

\[ \frac{\partial^2 u}{\partial x^2} + \varepsilon \frac{\partial^2 u}{\partial y^2} = f(x, y) \]

\[ \varepsilon = 0 \quad 5 \text{ point} \]

\[ \varepsilon = 1000 \quad 5 \text{ point} \]
AMG: Example

\[ \frac{\partial^2 u}{\partial x^2} + \varepsilon \frac{\partial^2 u}{\partial y^2} = f(x, y) \]

\[ \varepsilon < 1 \quad \varepsilon > 1 \]
RESULTS SINGLE CONTACT EHL

pressure

film

Fractional film content

footprint

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SINGLE CONTACT VALIDATION
STeady State

Standard mineral oil (shell TT9)
STEADY STATE

U=0.05 m/s

U=1.28 m/s

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TIME VARYING: LOAD


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Time Varying: ‘roughness’

h=280 nm

Venner, C.H., Kaneta, M., and Lubrecht, A.A.,
STARVED CONTACTS: EXPERIMENTAL
STARVED CONTACTS
STARVED CONTACTS

Direct relation between inlet layer and film thickness in the contact.

Accurate prediction when oil layer thickness correctly modeled.

SINGLE OIL LUBRICATED CONTACT

- Quick numerical solution allowing advanced studies
- Accurate prediction steady state, transient, roughness
- Simple Engineering models (Amplitude reduction)

- Also for starved contacts provided “inlet layer” is known
APPLICATION TO REAL BEARINGS?

Complications:

- Lubricated with grease (model as starved contact)
- Repeated overrolling in very short time
- Billions of overrollings in life-time !!!! (even MG doesn’t help enough)
- Lubricant migration (grease bleeding, cage, centrifugal forces etc.) determines inlet layer of oil on surface to each the contact
- .......

Solution: Thin Layer flow model for layer flow, linked to direct relation between layer and film from starved contact.
To develop a model that predicts change supply layer thickness.
Use model to predict long term film thickness decay.
THIN LAYER FLOW

1. Scale the N-S equations
2. Take the limit as taking the limit of as $\varepsilon \to 0$
3. Derive equation velocities
4. Insert the velocities into continuity equation.

Navier-Stokes equation (incompressible flow, constant viscosity):

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = f_x - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)
\]

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = f_y - \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)
\]

\[
\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = f_z - \frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)
\]
THIN LAYER FLOW

1. Scale the N-S equations
2. Take the limit as taking the limit of as $\varepsilon \to 0$
3. Derive equation velocities
4. Insert the velocities into continuity equation.

Step 1: $$\varepsilon = \frac{H}{L}, \quad W = \varepsilon U$$

$$\varepsilon^2 \text{Re} \left( \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} \right) = \bar{f}_x - \frac{\partial \bar{p}}{\partial x} + \varepsilon^2 \frac{\partial^2 \bar{u}}{\partial x^2} + \varepsilon^2 \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial^2 \bar{u}}{\partial z^2}$$

$$\varepsilon^2 \text{Re} \left( \frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} + \bar{w} \frac{\partial \bar{v}}{\partial z} \right) = \bar{f}_y - \frac{\partial \bar{p}}{\partial y} + \varepsilon^2 \frac{\partial^2 \bar{v}}{\partial x^2} + \varepsilon^2 \frac{\partial^2 \bar{v}}{\partial y^2} + \frac{\partial^2 \bar{v}}{\partial z^2}$$

$$\varepsilon^4 \text{Re} \left( \frac{\partial \bar{w}}{\partial t} + \bar{u} \frac{\partial \bar{w}}{\partial x} + \bar{v} \frac{\partial \bar{w}}{\partial y} + \bar{w} \frac{\partial \bar{w}}{\partial z} \right) = \bar{f}_z - \frac{\partial \bar{p}}{\partial z} + \varepsilon^4 \frac{\partial^2 \bar{w}}{\partial x^2} + \varepsilon^4 \frac{\partial^2 \bar{w}}{\partial y^2} + \varepsilon^2 \frac{\partial^2 \bar{w}}{\partial z^2}$$
**THIN LAYER FLOW**

1. Scale the N-S equations
2. Take the limit as taking the limit of as $\varepsilon \to 0$
3. Derive equation velocities
4. Insert the velocities into continuity equation.

Step 2:

$$\varepsilon = \frac{H}{L} \quad W = \varepsilon U$$

\[ \varepsilon^2 \text{Re} \left( \frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} \right) = \bar{f}_x - \frac{\partial \bar{p}}{\partial x} + \varepsilon^2 \frac{\partial^2 \bar{u}}{\partial x^2} + \varepsilon^2 \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial^2 \bar{u}}{\partial z^2} \]

\[ \varepsilon^2 \text{Re} \left( \frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} + \bar{w} \frac{\partial \bar{v}}{\partial z} \right) = \bar{f}_y - \frac{\partial \bar{p}}{\partial y} + \varepsilon^2 \frac{\partial^2 \bar{v}}{\partial x^2} + \varepsilon^2 \frac{\partial^2 \bar{v}}{\partial y^2} + \frac{\partial^2 \bar{v}}{\partial z^2} \]

\[ \varepsilon^4 \text{Re} \left( \frac{\partial \bar{w}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{w}}{\partial x} + \bar{v} \frac{\partial \bar{w}}{\partial y} + \bar{w} \frac{\partial \bar{w}}{\partial z} \right) = \bar{f}_z - \frac{\partial \bar{p}}{\partial z} + \varepsilon^4 \frac{\partial^2 \bar{w}}{\partial x^2} + \varepsilon^4 \frac{\partial^2 \bar{w}}{\partial y^2} + \varepsilon^2 \frac{\partial^2 \bar{w}}{\partial z^2} \]
THIN LAYER FLOW

1. Scale the N-S equations
2. Take the limit as taking the limit of as $\varepsilon \rightarrow 0$
3. Derive equation velocities
4. Insert the velocities into continuity equation.

Step 2:

$$0 = f_x - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial z^2} \right)$$

$$0 = f_y - \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial z^2} \right)$$

$$0 = f_z - \frac{\partial p}{\partial z}$$
THIN LAYER FLOW

1. Scale the N-S equations
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$$0 = f_z - \frac{\partial p}{\partial z}$$
THIN LAYER FLOW

1. Scale the N-S equations
2. Take the limit as taking the limit of as $\varepsilon \to 0$
3. Derive equation velocities
4. Insert the velocities into continuity equation.

Step 3:

$$p = f_z(z - h) - \tau_c \kappa + p_0$$

$$\langle u \rangle = \frac{1}{h} \int_0^h u \, dz = \frac{h^2}{3\mu} \left[ f_x + \frac{3}{8} h \frac{\partial f_z}{\partial x} + f_z \frac{\partial h}{\partial x} + \tau_s \left( \frac{\partial^3 h}{\partial x^3} + \frac{\partial^3 h}{\partial y^2 \partial x} \right) \right]$$

$$\langle v \rangle = \frac{1}{h} \int_0^h v \, dz = \frac{h^2}{3\mu} \left[ f_y + \frac{3}{8} h \frac{\partial f_z}{\partial y} + f_z \frac{\partial h}{\partial y} + \tau_s \left( \frac{\partial^3 h}{\partial x^2 \partial y} + \frac{\partial^3 h}{\partial y^3} \right) \right]$$
THIN LAYER APPROXIMATION

1. Scale the N-S equations
2. Take the limit as taking the limit of as $\varepsilon \to 0$
3. Derive equation velocities
4. Insert the velocities into continuity equation.

Step 4:

$$\frac{1}{3\mu} \frac{\partial}{\partial x} \left( h^3 \left[ f_x + \frac{1}{6} h \frac{\partial f_z}{\partial x} + f_z \frac{\partial h}{\partial x} + \tau_s \left( \frac{\partial^3 h}{\partial x^3} + \frac{\partial^3 h}{\partial y^2 \partial x} \right) \right] \right) + \ldots$$

$$\frac{1}{3\mu} \frac{\partial}{\partial y} \left( h^3 \left[ f_y + \frac{1}{6} h \frac{\partial f_z}{\partial y} + f_z \frac{\partial h}{\partial y} + \tau_s \left( \frac{\partial^3 h}{\partial x^3} + \frac{\partial^3 h}{\partial y^2 \partial x} \right) \right] \right) + \frac{\partial h}{\partial t} = 0$$


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1. Scale the N-S equations
2. Take the limit as taking the limit of as $\varepsilon \rightarrow 0$
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Step 4:

$$\frac{1}{3\mu} \frac{\partial}{\partial x} \left( h^3 \left[ f_x + \frac{3}{8} h \frac{\partial f_z}{\partial x} + f_z \frac{\partial h}{\partial x} + \tau_s \left( \frac{\partial^3 h}{\partial x^3} + \frac{\partial^3 h}{\partial y^2 \partial x} \right) \right] \right) + \ldots$$

$$\frac{1}{3\mu} \frac{\partial}{\partial y} \left( h^3 \left[ f_y + \frac{3}{8} h \frac{\partial f_z}{\partial y} + f_z \frac{\partial h}{\partial y} + \tau_s \left( \frac{\partial^3 h}{\partial x^2 \partial y} + \frac{\partial^3 h}{\partial y^3} \right) \right] \right) + \frac{\partial h}{\partial t} = 0$$


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THIN LAYER FLOW IN BEARINGS

Contact pressure effect

Centrifugal effect

Lubricant film thickness distribution

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2D ≠ 1D:

- Equipartition
- Contact pressure smoothening
- Surface tension

\[
\frac{1}{3\mu} \frac{\partial}{\partial y} \left( h^3 f_x \right) + \frac{\partial h}{\partial t} = 0
\]

- Hyperbolic equation, easily solved by method of characteristics

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CENTRIFUGAL EFFECTS RACEWAY

Example
CENTRIFUGAL EFFECTS RACEWAY

Example
Flow equation
\[
\frac{1}{r} \frac{\partial}{\partial s} \left( \frac{h^3}{3\eta_0} r \frac{f_s}{s} \right) + \frac{\partial h}{\partial t} = 0
\]

Body force equation
\[
f_s = \rho \Omega^2 r \frac{dr}{ds}
\]
CENTRIFUGAL EFFECT ROLLER

Flow equation

\[ \frac{1}{r} \frac{\partial}{\partial s} \left( \frac{h^3}{3 \eta_0} r f_s \right) + \frac{\partial h}{\partial r} = 0 \]

Body force equation

Raceways:

\[ f_{s, rw} = \rho \Omega_{r_w}^2 r \frac{dr}{ds} \]

Rollers:

\[ f_{s, rol} = \rho \Omega_{ca}^2 \left( \sin^2(\gamma) z_{rol} + \sin(\gamma) R_{ rol} \right) \frac{dz_{rol}}{ds} \]

\[ + \left( \left( \frac{1}{2} \cos^2(\gamma) + \frac{3}{2} \right) \Omega_{ca}^2 + 2 \Omega_{ca} \Omega_{rol} \cos(\gamma) + \Omega_{rol}^2 \right) \rho \frac{dr_{rol}}{ds} \]
COMBINING LAYERS: EQUIPARTION

The concept

Measurement setup

Measurements have been carried out by H. de Ruig and R. Meeuwenoord at SKF ERC

Measurement Results

Measurements have been carried out by H. de Ruig and R. Meeuwenoord at SKF ERC

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CENTRIFUGAL EFFECT: BEARING

- Time steps:

\[ t/\tau = \frac{\eta}{H^2 \rho \Omega^2} = 0, 0.3, 1, 3, 10, 50 \]

![Graphs showing flow type 1 and flow type 2](image)

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CONTACT PRESSURE: BEARING

Mass conservation

\[ \frac{\partial \rho \dot{h}}{\partial t} = - \frac{1}{\rho_0 l_t} \frac{\partial \hat{q}_y}{\partial y} \]

Mass flow in EHL contacts

\[ \hat{q}_y(y, t) = \sum_{k=1}^{n_c} \hat{q}_{y,k} \]

\[ \hat{q}_{y,k}(y, t) = \frac{1}{2\pi} \int_0^{a^+} \int_0^{a^-} \left( -\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial y} \right)_k \, dx \, d\psi \]

\[ \eta = \eta(p) \quad p = p(x, y, \psi, t) \]

\[ \rho = \rho(p) \quad h = h(x, y, \psi, t) \]

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CONNECTION TO “INSIDE CONTACT”

M=20.1, L = 10.4

Layer thickness

\[ \lim_{h_{oil} \to 0} h = \frac{2h_{\infty}^0}{\bar{\rho}} \]

Pressure

\[ \lim_{h_{oil} \to 0} p = p_h \sqrt{1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2} \]
SINGLE CONTACT: VALIDATION

Circular contact

Elliptical contact

F = 20 N, $p_h = 0.5$ GPa, $\eta_0 \approx 0.8$ Pa.s

F = 30 N, $p_h = 0.33$ GPa, $\eta_0 \approx 0.85$ Pa.s


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SINGLE CONTACT: VALIDATION

Starved Elasto-Hydrodynamic Lubrication

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SINGLE CONTACT: VALIDATION
CONTACT PRESSURE: BEARING

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CONTACT PRESSURE: BEARING

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VARYING BEARING LOAD

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VARYING BEARING SPEED

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FILM THICKNESS DECAY

\[ F_r = 10 \text{ kN}, \Omega_c = 3000 \text{ rpm} \]

<table>
<thead>
<tr>
<th>Load [kN]</th>
<th>Speed [rpm]</th>
<th>At [h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>750</td>
<td>2.5</td>
</tr>
<tr>
<td>10</td>
<td>1500</td>
<td>3.8</td>
</tr>
<tr>
<td>10</td>
<td>3000</td>
<td>5.8</td>
</tr>
<tr>
<td>5</td>
<td>3000</td>
<td>5.7</td>
</tr>
<tr>
<td>2.5</td>
<td>3000</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Deep groove Ball Bearing

Spherical Roller Bearing

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CONCLUSION

Film decay model for bearings developed from based on:
- Thin layer flow model
- Starved EHL

Model is developed to predict change of supply layer.
- Centrifugal effects
- Contact pressure effects

Model is validated experimentally.
CONCLUSION

- Larger layer thickness decay for ball bearing
- Decay depends on speed.
- Decay depends weekly on the load.
- Decay periods are short and significant lubricant supply to the track.
CONCLUSION

β Include lubricant supply mechanisms.
β Layer smoothening.
β Comparison with bearing tests: qualitative/quantitative

β …..AND MANY OTHER INTERESTING THINGS…..